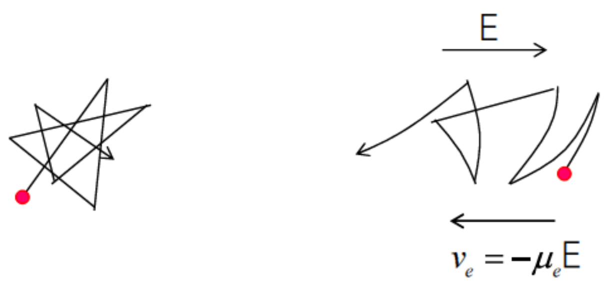
2 Electric Conduction

2.1 Drift Current

(1) Carrier drift operation and mobility

Carriers (conduction electrons and electron holes) in a crystal are affected by collisions with and scattering from the crystal lattice (roughly, the atoms that make up the crystal), and when the crystal lattice is perfectly periodic, this effect can be considered to be incorporated by the fact that the mass of the conduction electrons is the effective mass m^* (as discussed in Section 1.4). However, when the integrity of the crystal is impaired by thermal vibrations of lattice points or the presence of impurity atoms, i.e., when the periodicity of the lattice is disturbed, the carriers are affected by collisions and scattering at the (disturbed) location, and extra random movement is observed as shown in Figure 2.1 (a) and (b).



(a) No external force (b) Drift operation due to electric field

Figure 2.1 Random movement of carriers (conduction electrons)

Figure (a) shows the case where is no external force, and (b) is the case of drift operation under the force of an electric field. Such random motion of carriers is observed as noise. Particularly, those caused by thermal vibration of the lattice are the source of thermal noise. Note that the mass remains the effective mass m^* since the carrier is also affected by the original (periodic) lattice.

Consider the case where a carrier of charge q ($q = -e$ for conduction electrons, $q = e$ for electron holes) and effective mass m^* (m_e^* for conduction electrons, m_d^* for electron holes) drifts through the crystal under the force F due to the electric field Ｅ. Figure 2.2 shows the analytical model for the drift operation including the case of a perfectly periodic crystal lattice.

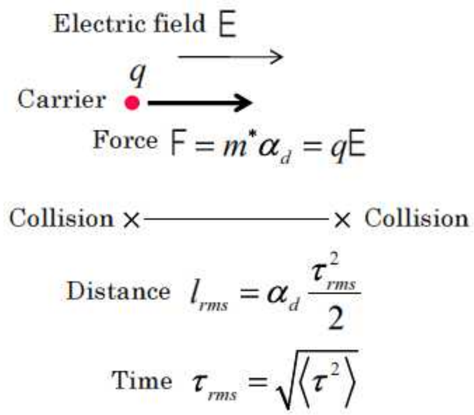


Figure 2.2　Analytical model for drift operation

The average time until the next collision or scattering of a carrier that has collided or scattered with a lattice or impurity is expressed as $\tau_{rms} = \sqrt{\langle \tau^2 \rangle}$. When an electric field Ｅ is applied, the carrier receives a force F due to the electric field while moving randomly, and an acceleration α_d is generated in the direction of the electric field. This behavior is expressed by the following equation

$$F = m^* \alpha_d = qE \tag{2.1}$$

The distance l_{rms} moved in the electric field direction in time τ_{rms} by the above acceleration α_d is given by the following equation.

$$l_{rms} = \alpha_d \frac{\tau_{rms}^2}{2} \tag{2.2}$$

Since it moves a distance l_{rms} in time τ_{rms}, the drift velocity v_d in the electric field direction can be expressed as follows.

$$v_d = \frac{l_{rms}}{\tau_{rms}} = \alpha_d \frac{\tau_{rms}^2}{2} \frac{1}{\tau_{rms}} = \frac{q\tau_{rms}}{2m^*}\mathsf{E} \tag{2. 3}$$

When the carrier is a conduction electron, replacing the variables $v_d \to v_e$, $q \to -e$, $m^* \to m_e^*$, and $\tau_{rms} \to \tau_e$, the above equation (2.3) becomes

$$v_e = \frac{-e\tau_e}{2m_e^*}\mathsf{E} = -\mu_e \mathsf{E} \qquad \text{(Drift velocity of conduction electrons)} \tag{2. 4}$$

$$\mu_e = \frac{e\tau_e}{2m_e^*} \qquad \text{(Mobility of conduction electrons)} \tag{2. 5}$$

When the carrier is an electron hole, replacing the variables $v_d \to v_h$, $q \to e$, $m^* \to m_h^*$, and $\tau_{rms} \to \tau_h$, the above equation (2.3) becomes

$$v_h = \frac{e\tau_h}{2m_h^*}\mathsf{E} = \mu_h \mathsf{E} \qquad \text{(Drift velocity of electron holes)} \tag{2. 6}$$

$$\mu_h = \frac{e\tau_h}{2m_h^*} \qquad \text{(Mobility of electron holes)} \tag{2. 7}$$

Table 2.1 shows examples of carrier mobility in intrinsic semiconductors. And Figure 2.3 shows examples of the drift velocity of conduction electrons in intrinsic semiconductors.

Table 2.1 Carrier mobility in intrinsic semiconductors

	Si	*Ge*	*GaAs*	*InAs*	*InP*	*GaN*	*SiC*
$\mu_e (m^2 / V \cdot s)$	0.15	0.38	0.88	3.30	0.46	0.12 †	0.1 †
$\mu_h (m^2 / V \cdot s)$	0.05	0.21	0.04	0.05	0.02	–	–

(unmarked data are based on Ref. [4], p.175, and † marked data are based on Ref. [14], p.24)

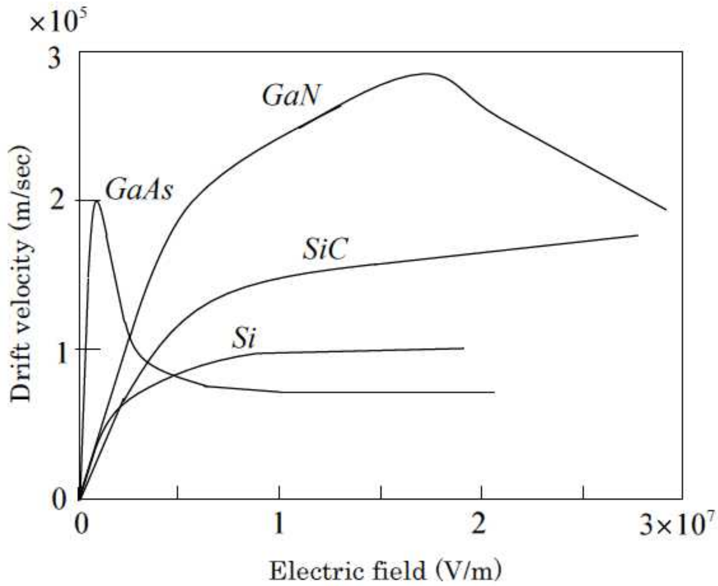


Figure 2.3 Examples of the drift velocity of conduction electrons in intrinsic semiconductors (according to Ref. [14], p.104)

While the electric field E is small, the mobility μ_e is constant and the drift velocity v_e increases in proportion to E. However, as E increases, v_e becomes a saturating property. Rough interpretations for velocity saturation include the following. The energy given by the electric field causes the electrons to become hot electrons, and the vibration of the electrons themselves increases (i.e., their temperature rises). In addition, the energy transferred from the electrons when they collide with the lattice further increases the vibration of the lattice. In other words, the energy given by the electric field is used to increase the vibrational energy of the electrons and the lattice. As a result, the frequency of electron collisions with the crystal lattice increases and τ_e becomes small. As a result, the mobility μ_e also becomes small and velocity saturation occurs.

(2) Drift current and conductivity

Drift current flows due to carrier drift operation. Figure 2.4 shows the drift current in the case where the carrier is a conduction electron.

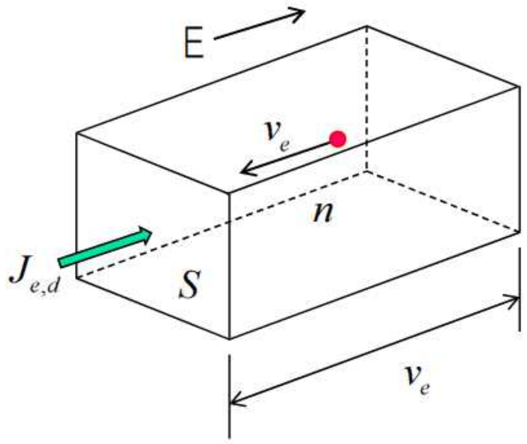


Figure 2.4 Drift current in the case where the carrier is a conduction electron.

When an electric field E is applied to an electron with charge $-e$, the electron moves in the opposite direction of the electric field with a drift velocity v_e. Here, the conduction electron density is n, the cross-sectional area is S, and using equation (2.4) for v_e, the drift current $I_{e,d}$ is given by

$$I_{e,d} = -env_eS = en\mu_e \mathrm{E} S \tag{2.8}$$

The drift current $I_{e,d}$ flows in the direction of the electric field E, and its value increases as the drift velocity v_e (mobility μ_e), conduction electron density n, and cross-sectional area S increase. From Equation (2.8), the drift current density (drift current through the unit cross-section) $J_{e,d}$ is

$$J_{e,d} = \frac{I_{e,d}}{S} = -env_e = en\mu_e \mathrm{E} = \sigma_n \mathrm{E} = \frac{1}{\rho_n}\mathrm{E} \tag{2.9}$$

$$\sigma_n = \frac{1}{\rho_n} = en\mu_e \tag{2.10}$$

where σ_n is conductivity and ρ_n is resistivity.

In the case of an N-type semiconductor with impurity (donor) density N_D, since $N_D \approx n$ at room temperature, the following equation is obtained.

$$\sigma_n = \frac{1}{\rho_n} \approx eN_D\mu_e \tag{2. 44}$$

The conductivity increases as N_D is increased. However, at low temperatures, the conductivity is small (resistivity is large) because the conduction electron density n is small (as discussed in section 1.7).

When the carrier is an electron hole, in the same way, the drift current $I_{h,d}$, the drift current density $J_{h,d}$, and the conductivity σ_p (resistivity ρ_p) can be obtained. The results are shown below.

$$J_{h,d} = \frac{I_{h,d}}{S} = epv_h = ep\mu_h \mathrm{E} = \sigma_p \mathrm{E} = \frac{1}{\rho_p}\mathrm{E} \tag{2. 12}$$

$$\sigma_p = \frac{1}{\rho_p} = ep\mu_h \tag{2. 13}$$

In the case of a P-type semiconductor with impurity (acceptor) density N_A, since $N_A \approx p$ at room temperature, the following equation is obtained.

$$\sigma_p = \frac{1}{\rho_p} = ep\mu_h \approx eN_A\mu_h \tag{2. 14}$$

However, since the electron hole density p is smaller at low temperatures, the conductivity is smaller (the resistivity is larger), which occurs in the same way.

2.2 Diffusion Current

If the carrier density differs from place to place for some reason, such as carrier injection, the carriers will try to diffuse from a place of higher density to a place of lower density (law of increasing entropy). As shown in Figure 2.5, when carriers are injected from the point $x = 0$, they diffuse in the x -axis plus direction. The number of carriers N_q passing through the unit cross-section in unit time is given by the following equation using the carrier density $n_q(x)$ and the carrier diffusion coefficient D_q.

$$N_q = -D_q \frac{dn_q(x)}{dx} \tag{2.15}$$

From this, the diffusion current density $J_{q,s}$ due to carrier diffusion is given by

$$J_{q,s} = qN_q = -qD_q \frac{dn_q(x)}{dx} \tag{2.16}$$

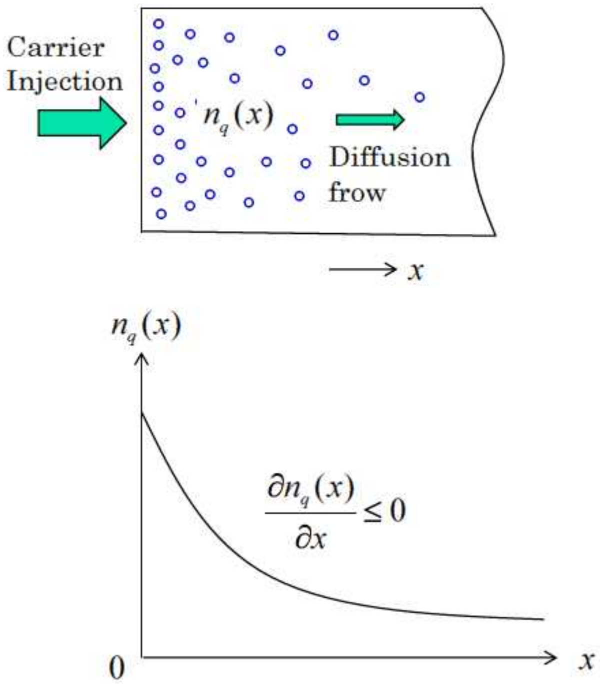


Figure 2.5　Carrier diffusion when carriers are injected

The diffusion current density when the carriers are conduction electrons is given by substituting the variables $q \to -e$, $n_q(x) \to n(x)$, $D_q \to D_e$, and $J_{q,s} \to J_{e,s}$ in the above equation (2.16) as follows

$$J_{e,s} = eD_e \frac{dn(x)}{dx} \tag{2. 17}$$

On the other hand, the diffusion current density when the carriers are electron holes is given by substituting the variables $q \to e$, $n_q(x) \to p(x)$, $D_q \to D_h$, and $J_{q,s} \to J_{h,s}$ in the above equation (2.16) as follows

$$J_{h,s} = -eD_h \frac{dp(x)}{dx} \tag{2. 18}$$

The diffusion coefficient introduced here is related to the mobility described in section 2.1 by the following equation (Einstein's relation, see Appendix D for derivation)

$$\frac{D_e}{\mu_e} = \frac{D_h}{\mu_h} = \frac{k_B T}{e} \tag{2. 19}$$

2.3 Generation and Recombination of Carriers and Continuity Equation

Temperature, light, and other energies excite electrons in the valence band to generate conduction electrons and electron holes. This is called pair generation of electrons and holes. In contrast, the process in which a conduction electron and an electron hole combine, releasing energy and annihilating the conduction electron and electron hole is called recombination. In addition to direct recombination, in which conduction electrons and electron holes bond directly, there are indirect recombination, in which they bond via impurity levels in the semiconductor, and surface recombination, in which they bond via surface energy levels on the semiconductor surface. Here, we refer to them collectively as simply recombination, regardless of their type.

Here, we discuss the continuity equation that takes into account pair generation and recombination of carriers in semiconductors. Figure 2.6 shows the analytical model used to derive the equations. Here, we consider the one-dimensional flow of electron holes (minority carriers) in an N-type semiconductor.

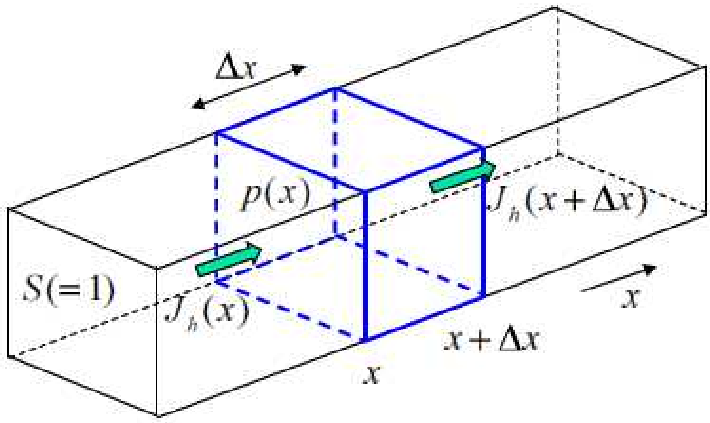


Figure 2.6 Analytical model used to derive the continuity equation

Let $S=1$ (unit cross section) and consider the change in the number of electron holes in the region bounded by $x \sim x+\Delta x$ (called the box). Let $p(x)$ be the electron hole density (number of holes per unit volume) at x and $J_h(x)$ be the current density due to electron holes. The number of electron holes flowing into the box in unit time in the form of current is $\frac{1}{e}J_h(x)$, and the number of electron holes leaving the box in unit time in the form of current is $\frac{1}{e}J_h(x+\Delta x)$. From this, the number of electron holes leaving the box in the form of current in time Δt is given by

$$\left(\frac{1}{e}J_h(x+\Delta x)-\frac{1}{e}J_h(x)\right)\Delta t=\frac{1}{e}\frac{dJ_h(x)}{dx}\Delta x\Delta t \tag{2.20}$$

Next, consider the change in the number of electron holes in the box due to carrier pair generation and recombination. If the probability that an electron hole is annihilated by recombination in a unit time is R_h, then R_h is related to the lifetime of an electron hole τ_h (the time between its creation by pair generation and its annihilation by recombination) by the following equation

$$R_h = \frac{1}{\tau_h} \tag{2. 21}$$

From this, the number of electron holes annihilated (by recombination) in the box in Δt time is given by

$$R_h p(x)\Delta x \Delta t = \frac{1}{\tau_h} p(x)\Delta x \Delta t \tag{2. 22}$$

If the probability of generating one electron hole per unit time is G_h then at thermal equilibrium, the number of electron holes generated equals the number of electron holes annihilated, and the following relationship holds.

$$G_h p_0 \Delta x \Delta t = R_h p_0 \Delta x \Delta t = \frac{1}{\tau_h} p_0 \Delta x \Delta t \tag{2. 23}$$

p_0 is the electron hole density at thermal equilibrium (number of electron holes per unit volume)

From this, the net number of electron holes annihilated in the box in time Δt due to pair generation and recombination when not in thermal equilibrium is given by

$$R_h p(x)\Delta x \Delta t - G_h p_0 \Delta x \Delta t = \frac{p(x) - p_0}{\tau_h} \Delta x \Delta t \tag{2. 24}$$

From the above, the number of electron holes lost from the box during Δt time due to current and generation/recombination is given by

$$-\frac{dp(x)}{dt}\Delta x\Delta t = \frac{1}{e}\frac{dJ_h(x)}{dx}\Delta x\Delta t + \frac{p(x)-p_0}{\tau_h}\Delta x\Delta t \tag{2.25}$$

$$\longrightarrow \quad -\frac{dp(x)}{dt} = \frac{1}{e}\frac{dJ_h(x)}{dx} + \frac{p(x)-p_0}{\tau_h} \tag{2.26}$$

The above equation (2.26) gives an expression for the continuity for electron holes, taking into account current and generation and recombination. Now consider the case where there is no electric field ($E = 0$), the only current is the diffusion current, and the steady state ($\frac{dp(x)}{dt} = 0$). Using Eq. (2.18) for the diffusion current, the above Eq. (2.26) becomes

$$-\frac{dp(x)}{dt} = 0 = -D_h\frac{d^2 p(x)}{dx^2} + \frac{p(x)-p_0}{\tau_h} \tag{2.27}$$

From the above, the following equation is obtained for electron holes (minority carriers).

$$0 = -D_h\frac{d^2 p^*(x)}{dx^2} + \frac{p^*(x)}{\tau_h} \tag{2.28}$$

$$p^*(x) = p(x) - p_0 \qquad \text{(excess electron hole density)} \tag{2.29}$$

p_0 is the electron hole density at thermal equilibrium and is independent of location, whereas $p^*(x)$ is the difference from p_0 and is a location-dependent function. Assuming $L_h = \sqrt{\tau_h D_h}$ (electron hole diffusion distance note *3), Eq. (2.28) becomes

$$0 = p^*(x) - L_h^2\frac{d^2 p^*(x)}{dx^2} \tag{2.30}$$

From this, the general solution for $p^*(x)$ is

$$p^*(x) = C_1 \exp\left(\frac{-x}{L_h}\right) + C_2 \exp\left(\frac{x}{L_h}\right) \tag{2.31}$$

The boundary conditions are (I) $p^*(x) \to 0$ when $x \to \infty$, so $C_2 = 0$, also, (II) when Δp excess carriers are steadily injected in unit time from the left side of the plane set at $x = 0$, $p^*(0) = p(0) - p_0 = \Delta p$, so $C_1 = \Delta p$. From this, p(x) is given by

$$p^*(x) = \Delta p \exp\left(\frac{-x}{L_h}\right) \tag{2.32}$$

Figure 2.7 shows how electron holes (minority carriers) injected from the left side of the plane at $x = 0$ diffuse through the N-type semiconductor crystal.

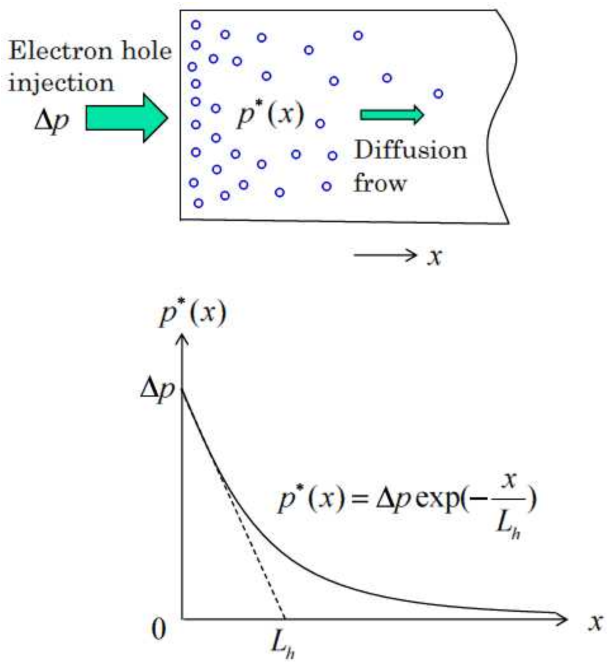


Figure 2.7 How electron holes (minority carriers) injected from $x = 0$ diffuse through an N-type semiconductor.

Note *3

The diffusion distance L_d is the average distance that a minority carrier (in this case an electron hole) travels by diffusion in a semiconductor crystal from the time it is generated (injected) to its annihilation. In contrast, l_{rms}, introduced in section 2.1, is the average distance a carrier travels (in the direction of the electric field) from the time it strikes one lattice to the next, and is generally $l_{rms} << L_d$.

Appendix D　Einstein's Equation

Consider an N-type semiconductor that is not externally connected and is in thermal equilibrium, and where the donor density varies with gradient distribution, as shown in Figure D.1.

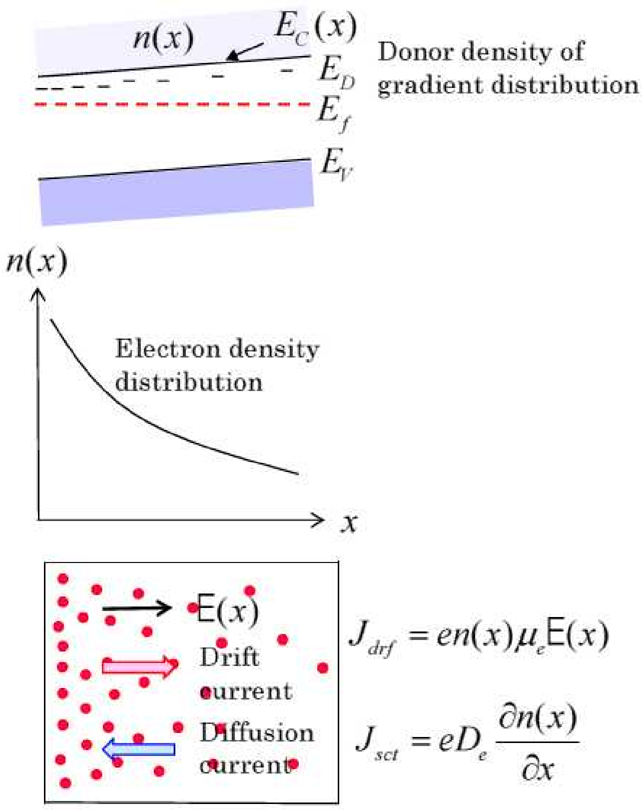


Figure D.1　Energy band structure, conduction electron density distribution, diffusion and drift currents when donor density varies with gradient distribution

In thermal equilibrium, the Fermi (energy) level E_f is constant regardless of location, so the energy $E_C(x)$ at the bottom of the conduction band varies with the donor density, and thus the conduction electron density $n(x)$ also varies with location. Conduction electrons try to diffuse from a place of higher density to a place of lower density. In thermal equilibrium without external connection, the diffusion current J_{sct} due to diffusion and the drift current J_{drf} due to the electric field cancel each other and no current flow is realized. In this case, the following equation holds.

$$J_{drf} + J_{sct} = en(x)\mu_e \mathsf{E}(x) + eD_e \frac{\partial n(x)}{\partial x} = 0 \tag{D.1}$$

By the way, $n(x)$ has the following relationship with the energy $E_C(x)$ at the bottom of the conduction band (from section 1.5)

$$n(x) = N_C \exp\left(-\frac{E_C(x) - E_f}{k_B T}\right) \tag{D.2}$$

Differentiating this yields the following

$$\frac{dn(x)}{dx} = -\frac{n(x)}{k_B T}\frac{d(E_C(x) - E_f)}{dx} = -\frac{n(x)}{k_B T}\frac{dE_C(x)}{dx} \tag{D.3}$$

Here, the force $F(x)$ working on the electron, the electric field $\mathsf{E}(x)$, and the energy $E_C(x)$ have the following relationship (see Appendix E in Chapter 3).

$$\mathsf{F}(x) = -e\mathsf{E}(x) = -\frac{dE_C(x)}{dx} \tag{D.4}$$

Substituting equations (D.3) and (D.4) into equation (D.1), we obtain

$$0 = n(x)\mu_e \frac{dE_C(x)}{dx} - eD_e \frac{n(x)}{k_B T}\frac{dE_C(x)}{dx} \tag{D.5}$$

From this we obtain the following relation (Einstein's equation) linking the diffusion coefficient and mobility.

$$\frac{D_e}{\mu_e} = \frac{k_B T}{e} \tag{D.6}$$

In the case where the carrier is an electron hole, the equation is as follows.

$$\frac{D_h}{\mu_h} = \frac{k_B T}{e} \tag{D.7}$$