3 PN Junction and Devices

3.1 PN Junction

(1) Physics of PN junction

Figure 3.1 shows the definition of the work function ϕ of a crystal. The figure shows the case of a metallic crystal. The work function ϕ is defined as the difference between the vacuum (energy) level E_{vac} and the Fermi (energy) level E_f, $\phi = E_{vac} - E_f$. It is the energy emitted when an electron falls from the vacuum level to the Fermi level in a crystal, and conversely, it is the energy required for an electron in the Fermi level of the crystal to jump to the vacuum (vacuum level). The work function ϕ and the Fermi level E_f correspond one-to-one.

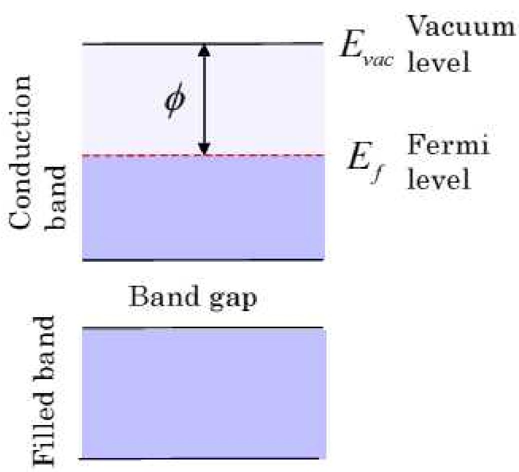


Figure 3.1 Definition of work function ϕ for crystals

Figure 3.2 shows the relationship between work function and electronegativity of metallic crystal materials. The larger the work function, the higher the electronegativity (easier to attract electrons). This can be considered to mean that the larger the work function, i.e., the lower the Fermi level, the lower the potential energy for electrons originally held by the crystal material.

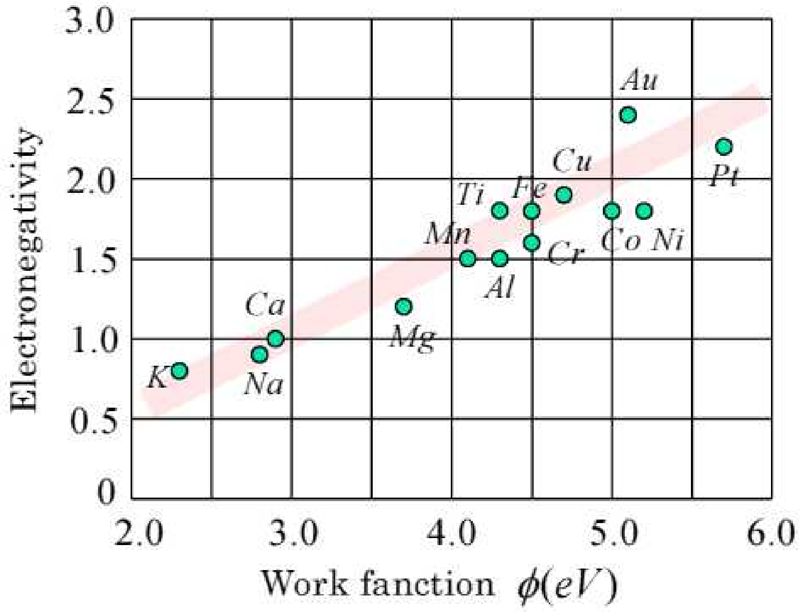
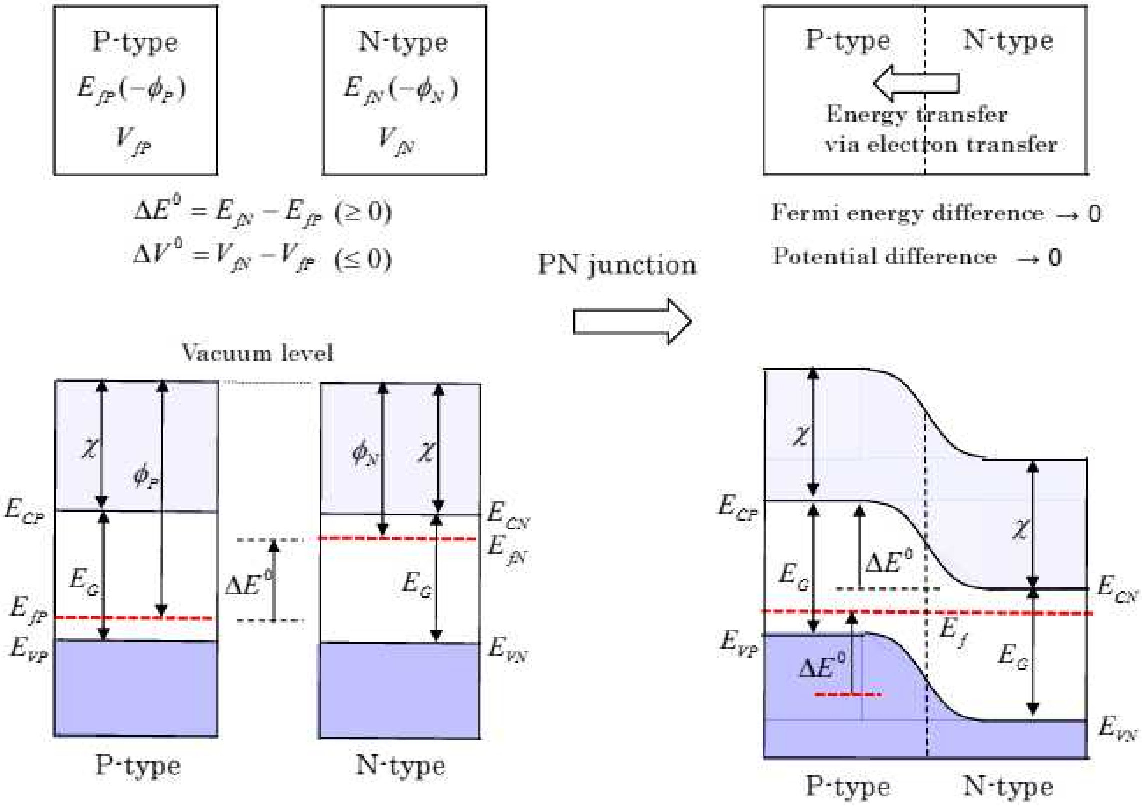


Figure 3.2 Work function and electronegativity of metal crystal materials (data based on Ref. [10], p. 321 and Ref. [11], p. 92)

What about semiconductors? Even in semiconductors, it can be considered that the lower the Fermi level E_f (the larger the work function ϕ), the lower the potential energy for the electrons held by the crystal. This means that P-type semiconductors have lower potential energy for electrons than N-type semiconductors, even for semiconductors of the same material.

Figure 3.3 shows the energy band structure of P- and N-type semiconductors (a) before and (b) after junction. The potential energy for electrons in N-type semiconductors is originally higher than in P-type semiconductors by $\Delta E^0 = E_{fN} - E_{fP}$. When P- and N-semiconductors with different potential energy are joined, electrons are transferred from the N-semiconductor with higher energy to the P-semiconductor with lower energy. This transfer continues until the energy difference that originally existed between the two semiconductors disappears and they are in the same energy state. That is, energy is transferred from the N-type semiconductor to the P-type semiconductor via electron transfer at the junction, and when the Fermi levels of the two semiconductors become equal, electron transfer stops and a state of thermal equilibrium is achieved. When the Fermi (energy) level changes, the energy band structure changes (curves) accordingly. In this case, the electron affinity χ and energy gap E_G do not change between N- and P-types as long as the same semiconductor, and also even when the energy band structure is changed (curved) by the junction, it also changes in a manner that the values of χ and E_G are conserved everywhere. Figure 3.3(b) shows the energy band structure after the junction. In the figure, the energy of the P-type semiconductor is depicted as ΔE^0 (≥ 0) changing (in the direction of higher energy). Note that the vacuum level and the energy level at the top of the conduction band are two different things. However, when drawing the band structure of a semiconductor junction in this document, the distinction between the two is not made.



(a) before junction (b) after junction

Figure 3.3 Energy band structure of P- and N-type semiconductors

The Fermi levels E_{fP} and E_{fN} of the P- and N-type semiconductors before the junction are expressed using the potentials V_{fP} and V_{fN} as follows (see Appendix E).

$$E_{fP} = -eV_{fP} \qquad E_{fN} = -eV_{fN} \tag{3.1}$$

From this, the following equation holds.

$$\Delta E^0 = E_{fN} - E_{fP} = -e(V_{fN} - V_{fP}) = -e\Delta V^0 \quad (\geq 0) \tag{3.2}$$

$$\Delta V^0 = V_{fN} - V_{fP} \quad (\leq 0)$$

At the time of junction, if the potentials of both semiconductors can be made equal due to electron transfer to cancel out the potential difference ΔV^0 that originally existed, then the Fermi levels of the two semiconductors will also be equal and a state of thermal equilibrium will be realized. Here, the change in potential at the junction is created by the electric charge generated at the junction.

-- Column 2 --

The definitions and units of potential energy, potential, voltage, and power are listed below for reference.

Energy

- Energy = Potential energy + Kinetic energy
- When considering the band structure of semiconductors in this document, only potential energy is considered and kinetic energy is ignored.

Potential energy

- The energy that is "stored" in an object when it is in a certain position
- The amount of work (energy) that a (charged) particle at a "certain position" can do when it moves to infinity (potential = 0).
- When potential energy is positive, energy is released when moving. When it is negative, energy must be given to move.)
- (In the case of potential energy due to gravity in mechanics, the amount of work that a (mass) particle can do when moving to infinity (gravity = 0) with zero velocity under gravity)

The unit is $[J]=[N \cdot m]=\left[Kg\dfrac{m^2}{sec^2}\right]=[C \cdot V]$

Potential

・The amount of work that a "unit charge" can do when it moves to infinity (potential = 0) with a velocity of 0

The unit is $\left[\dfrac{J}{C}\right]=\left[\dfrac{N \cdot m}{C}\right]=\left[Kg\dfrac{m^2}{\sec^2 \cdot C}\right]=[V]$

Voltage (= Potential difference)

・Voltage=Difference in potential between two points = Potential difference

The unit is $[V]=\left[\dfrac{J}{C}\right]=\left[\dfrac{N \cdot m}{C}\right]=\left[Kg\dfrac{m^2}{\sec^2 \cdot C}\right]$

Power

・Time average of energy = work per unit time

The unit is $[W]=\left[\dfrac{J}{\sec}\right]=\left[\dfrac{C \cdot V}{\sec}\right]=[A \cdot V]$

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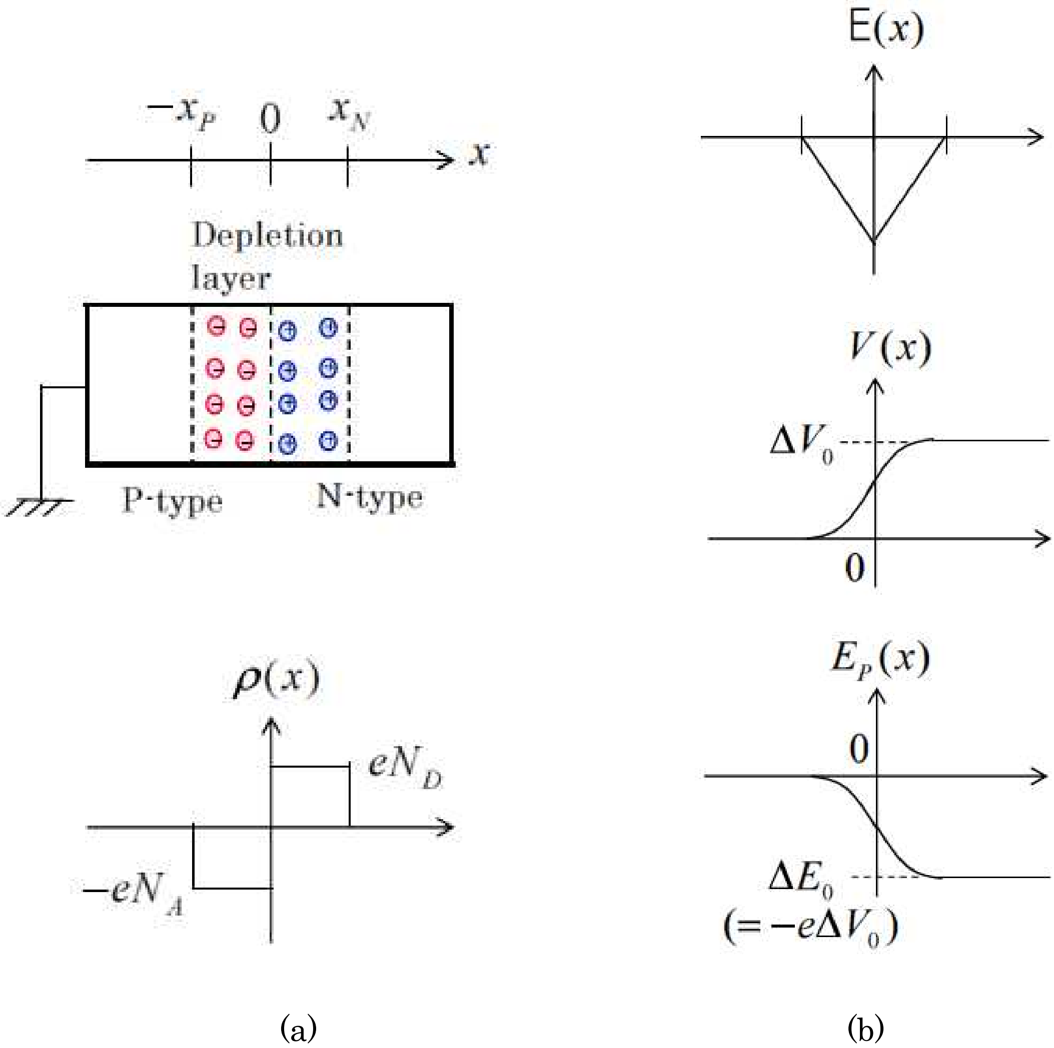
(2) Potential and energy band of PN junction

In the following, we will quantitatively analyze the potential and energy change caused by the electron transfer in the PN junction. Figure 3.4 shows (a) the analytical model and (b) the electric field $E(x)$, potential $V(x)$, and potential energy $E_P(x)$ of the PN junction. When P- and N-semiconductors are joined, conduction electrons from the N-semiconductor diffuse into the P-semiconductor and electron holes from the P-semiconductor diffuse into the N-semiconductor (the movement of electron holes (holes through which electrons have escaped) can also be said to be essentially the movement of electrons). As a result, negatively ionized acceptor atoms remain in the P-type semiconductor junction and positively ionized donor atoms remain in the N-type semiconductor junction. This area is called the depletion layer. Figure 3.4(a) shows this situation.



(a) (b)

Figure 3.4 (a) analytical model of a PN junction and (b) electric field E(x), potential $V(x)$ and potential energy $E_P(x)$

The electric field E(x) generated by the ionized acceptor and donor atoms creates a potential difference ΔV_0 between the P and N regions. If the potential difference ΔV_0 cancels the original potential difference ΔV^0, the Fermi (energy) levels of the two semiconductors also change so as to be $E_{fP} = E_{fN} = E_f$, as described below, and a state of thermal equilibrium is realized. If the coordinates are taken as in Figure 3.4(a) and the potential at x is $V(x)$ (where the potential is a one-dimensional function that varies only in the x direction), $V(x)$ must satisfy the following one-dimensional Poisson equation.

$$
\left.
\begin{aligned}
\frac{d^2V}{dx^2} &= \frac{eN_A}{\varepsilon_S} && \left(-x_p \le x \le 0\right) \\
\frac{d^2V}{dx^2} &= \frac{-eN_D}{\varepsilon_S} && \left(0 \le x \le x_N\right) \\
\frac{d^2V}{dx^2} &= 0 && \left(x \le -x_P\right), \left(x_N \le x\right)
\end{aligned}
\right\} \tag{3.3}
$$

ε_S is the dielectric constant of the semiconductor

N_A is the ionized acceptor density and N_D is the ionized donor density, which are constant values in the depletion layer

From the above equation (3.3), solving $\mathsf{E}(x) = -dV(x)/dx$ under the boundary conditions for the electric field, $\mathsf{E}(-x_P) = \mathsf{E}(x_N) = 0$, yields the following equation for $\mathsf{E}(x)$, where the direction of $\mathsf{E}(x)$ is expressed in the x-axis right direction as plus.

$$
\left.
\begin{aligned}
\mathsf{E}(x) &= -\frac{eN_A(x + x_P)}{\varepsilon_S} && \left(-x_p \le x \le 0\right) \\
\mathsf{E}(x) &= -\frac{eN_D(x_N - x)}{\varepsilon_S} && \left(0 \le x \le x_N\right) \\
\mathsf{E}(x) &= 0 && \left(x \le -x_P\right), \left(x_N \le x\right)
\end{aligned}
\right\} \tag{3.4}
$$

Furthermore, solving for $V(x)$ under the boundary conditions for potential, $V(-x_P) = 0$, yields the following equation.

$$
\left.
\begin{aligned}
V(x) &= 0 && \left(x \le -x_P\right) \\
V(x) &= \frac{eN_A}{2\varepsilon_S}(x + x_P)^2 && \left(-x_p \le x \le 0\right) \\
V(x) &= \frac{eN_D}{2\varepsilon_S}(-x^2 + 2x_N x) + \frac{eN_A}{2\varepsilon_S}x_P^2 && \left(0 \le x \le x_N\right) \\
V(x) &= \frac{e}{2\varepsilon_S}(N_D x_N^2 + N_A x_P^2) && \left(x_N \le x\right)
\end{aligned}
\right\} \tag{3.5}
$$

From this, the potential difference ΔV_0 across the junction is given by

$$\Delta V_0 = V(x_N) - V(-x_P) = \frac{e}{2\varepsilon_S}\left(N_D x_N^2 + N_A x_P^2\right) \qquad (\geq 0) \tag{3.6}$$

Due to the depletion layer charge, the potential of the N-type semiconductor is higher than that of the P-type semiconductor by ΔV_0. Furthermore, using the relationship between potential energy and potential (see Appendix E), the energy difference ΔE_0 between the N- and P-type semiconductors can be expressed by the following equation

$$\begin{aligned} \Delta E_0 &= E_P(x_N) - E_P(-x_P) \\ &= -e[V(x_N) - V(-x_P)] = -e\Delta V_0 \end{aligned} \qquad (\leq 0) \tag{3.7}$$

The energy of an N-type semiconductor changes ΔE_0 with respect to the energy of a P-type semiconductor (ΔE_0 is a negative value). The potential difference δV resulting from the addition of the potential difference ΔV_0 generated by the junction to the original potential difference ΔV^0 between the N- and P-type semiconductors can be expressed as follows.

$$\delta V = \Delta V^0 + \Delta V_0 \tag{3.8}$$

The energy difference δE_P resulting from the addition of the energy difference ΔE_0 caused by the junction to the original energy difference ΔE^0 can be expressed as follows.

$$\delta E_P = \Delta E^0 + \Delta E_0 = -e\Delta V^0 - e\Delta V_0 = -e\delta V \tag{3.9}$$

Here, when the potential difference ΔV_0 caused by the junction cancels out the original potential difference ΔV^0 and δV becomes 0 as in the following equation,

$$\delta V = \Delta V^0 + \Delta V_0 = 0 \quad \longrightarrow \quad \Delta V_0 = -\Delta V^0 \tag{3.10}$$

the energy difference δE_P also becomes 0 as follows.

$$\delta E_P = \Delta E^0 + \Delta E_0 = -e\delta V = 0 \quad \longrightarrow \quad \Delta E_0 = -\Delta E^0 \tag{3.11}$$

The junction changes the energy (i.e., Fermi (energy) level) of both semiconductors so that they are equal, resulting in a state of thermal equilibrium. With the change in Fermi level, the energy band structure of the semiconductors also changes (curves). Figure 3.3(b) above illustrates this.

-- Column 3 --

Figure 3.4 is often used to quantitatively analyze the energy band structure of a PN junction. According to this figure, a potential difference is generated between the P- and N-semiconductors due to the charge generated in the junction. Then, what happens if we measure the voltage between the PN-junction P-type and N-type semiconductors? Naturally, the voltage would be measured as 0 volts. Then, what does the potential difference (=voltage) due to the charge generated in the junction mean?

To explain this, we should consider that the potential energy values originally held by the P- and N-type semiconductors are different and that, as a result, the potentials originally existing in the two semiconductors are different. It can be assumed that the potential difference that originally existed is canceled by the potential difference (=voltage) created by the charge at the junction, resulting in a voltage of 0 volts, and that the difference in potential energy between the two semiconductors also disappears at that time. In fact, this potential difference (=voltage) originally existing at the junction is called the built-in voltage and is taken into account in the analysis.

In this document, however, we will express more clearly the idea of the potential energy that semiconductor crystals originally possess for electrons.

For this reason, the symbols for the original energy difference ΔE^0, which exists originally, and the energy difference ΔE_0, which was created at the time of joining to cancel ΔE^0, and also, the original potential difference ΔV^0, which exists originally, and the potential difference ΔV_0, which is produced in relation to the electric charge at the junction to cancel ΔV^0, are treated differently. This concept is also applied to other junctions (metal-semiconductor junctions, MOS junctions, and hetero junctions), which will be described later.

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(3) PN junction capacitance

Since the positive and negative charges due to ionized donors and acceptors in the depletion layer are equal at thermal equilibrium, the following equation holds.

$$x_P N_A = x_N N_D \tag{3.12}$$

From equations (3.6) and (3.12), we obtain

$$x_P = \sqrt{\frac{2\varepsilon_S N_D \Delta V_0}{eN_A(N_D + N_A)}} \qquad x_N = \sqrt{\frac{2\varepsilon_S N_A \Delta V_0}{eN_D(N_D + N_A)}} \tag{3.13}$$

From this, the depletion layer thickness is given by

$$L = x_P + x_N = \sqrt{\frac{2\varepsilon_S (N_D + N_A)\Delta V_0}{eN_A N_D}} \tag{3.14}$$

In addition, the charge per unit area accumulated in the depletion layer at thermal equilibrium is as follows.

$$
\left.\begin{aligned}
Q_P &= -ex_P N_A = -eN_A\sqrt{\frac{2\varepsilon_S N_D \Delta V_0}{eN_A(N_A+N_D)}} = -\sqrt{\frac{2\varepsilon_S eN_A N_D \Delta V_0}{(N_A+N_D)}} = -Q_0 \\
Q_N &= ex_N N_D = eN_D\sqrt{\frac{2\varepsilon_S N_A \Delta V_0}{eN_D(N_A+N_D)}} = \sqrt{\frac{2\varepsilon_S eN_A N_D \Delta V_0}{(N_A+N_D)}} = Q_0
\end{aligned}\right\} \tag{3.15}
$$

From this, the capacitance C_0 per unit area at thermal equilibrium can be obtained as follows.

$$
C_0 = \frac{dQ_0}{d\Delta V_0} = \frac{d}{d\Delta V_0}\sqrt{\frac{2\varepsilon_S eN_A N_D \Delta V_0}{(N_A+N_D)}} = \sqrt{\frac{\varepsilon_S eN_A N_D}{2(N_A+N_D)\Delta V_0}} = \frac{\varepsilon_S}{L} \tag{3.16}
$$

Equation (3.16) shows that the capacitance C_0 due to a depletion layer of thickness L in the PN junction is equivalent to a parallel plate capacitor of thickness L. This is shown in Figure 3.5.

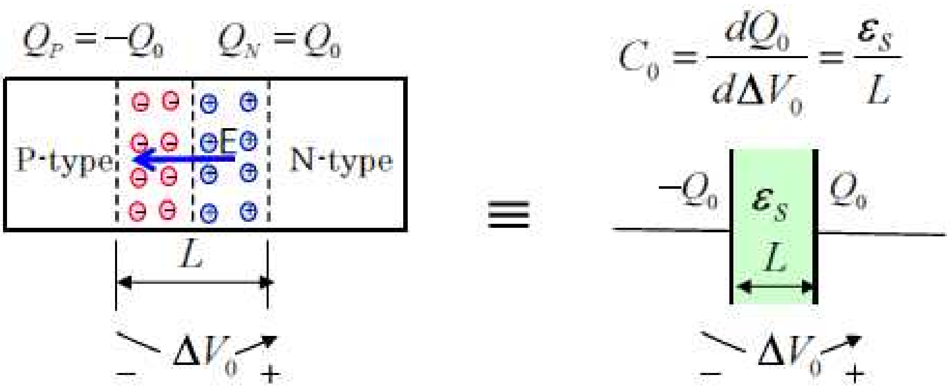

Figure 3.5 PN Junction Capacitance

3.2 PN Junction Diode

Consider a bias voltage V applied to a PN junction semiconductor as shown in Figure 3.6, where the N-type semiconductor side is grounded (potential is zero) and the P-type semiconductor side has a potential V.

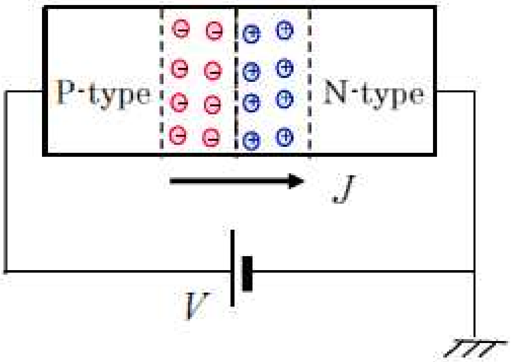

Figure 3.6　Applying a bias voltage V to the PN junction

In the following, we will examine the case of (a) forward bias ($V \geq 0$) and (b) reverse bias ($V \leq 0$) separately.

(a) In the case of forward bias ($V \geq 0$)

When a voltage V is applied to a P-type semiconductor, the Fermi (energy) level on the P-type semiconductor side changes by $\Delta E_V (= -eV)$ relative to the thermal equilibrium state (at $V \geq 0$, the energy changes in the lower (negative) direction). When the Fermi level changes ΔE_V, the energy band structure changes (curves) accordingly. In this case, the total energy difference ΔE_T and potential difference ΔV_T from the thermal equilibrium state before bonding are as follows.

$$\Delta E_T = \Delta E^0 + \Delta E_V = -\Delta E_0 + \Delta E_V = e(\Delta V_0 - V) \tag{3.17}$$

$$\longrightarrow \qquad \Delta V_T = V - \Delta V_0$$

where $\Delta E^0 = -\Delta E_0 = e\Delta V_0$, $\Delta E_V = -eV$, $\Delta E_T = -e\Delta V_T$

Figure 3.7 shows how the energy band structure changes when forward bias ($V \geq 0$) is applied. The figure also shows the relationship between ΔE^0, ΔE_V, and ΔE_T.

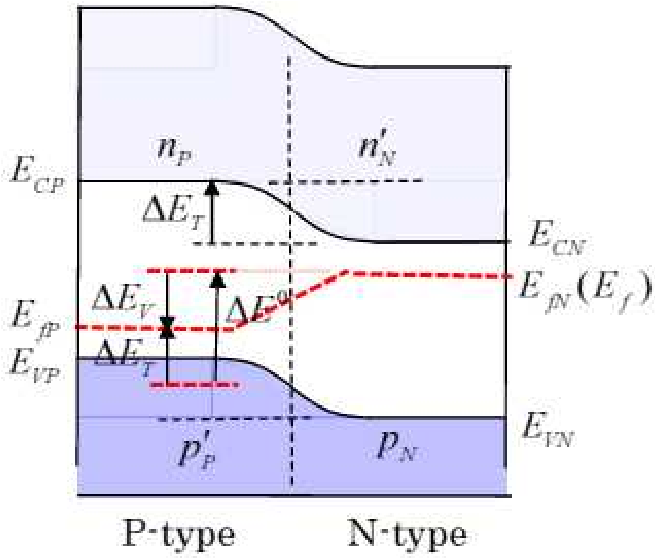

Figure 3.7 Energy band structure when forward bias ($V \geq 0$) is applied

When a voltage V is applied, the conduction electron density n'_N above the energy level E_{CP} in an N-type semiconductor is given by (see Appendix F)

$$n'_N = n_N \exp\left(-\frac{\Delta E_T}{k_B T}\right) = n_P \exp\left(\frac{eV}{k_B T}\right) \tag{3.18}$$

At this time, the following density of conduction electrons flow and diffuse from the N-type semiconductor to the P-type semiconductor.

$$n'_N - n_P = n_P\left(\exp\left(\frac{eV}{k_B T}\right) - 1\right) \tag{3.19}$$

Similarly, the electron hole density p'_P below the energy level E_{VN} in a P-type semiconductor is given by

$$p'_P = p_P \exp\left(-\frac{\Delta E_T}{k_B T}\right) = p_N \exp\left(\frac{eV}{k_B T}\right) \tag{3.20}$$

In this case, the following density of electron holes will flow and diffuse from the P-type semiconductor to the N-type semiconductor.

$$p'_P - p_N = p_N \left(\exp\left(\frac{eV}{k_B T} \right) - 1 \right) \tag{3.21}$$

When conduction electrons (minority carriers) flow into a P-type semiconductor and diffuse through it, a diffusion current due to conduction electrons flows. In addition, electron holes (minority carriers) flow into an N-type semiconductor and diffuse through it, resulting in a diffusion current due to electron holes. Below, we obtain these diffusion currents. In order to avoid the complexity of the analysis, we will use the coordinates shown in Figure 3.8, neglecting the dimensions of the depletion layer portion as an approximation.

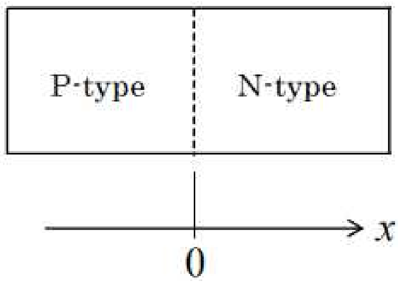

Figure 3.8 Coordinates of PN junction used in analysis

Here, we first analyze the electron holes (minority carriers) that flow into the N-type semiconductor. When the electron hole density at thermal equilibrium in an N-type semiconductor is p_{N0}, the electron hole density at coordinate point $x(\geq 0)$ is $p_N(x)$, and the excess electron hole density is $p_N^*(x)$, these are related by the following equation.

$$p_N^*(x) = p_N(x) - p_{N0} = p_N(x) - p_N \tag{3.22}$$

The last equation above shows that the electron hole density p_{N0} at thermal equilibrium for an N-type semiconductor is the same as the p_N used in equations (3.20) and (3.21). Excess electron holes diffuse into the N-type semiconductor and recombine with conduction electrons, which are the majority carriers of the N-type semiconductor. In the steady state, electron holes are constantly supplied from the P-type semiconductor side, and conduction electrons that recombine with them are continuously supplied from the ground side to the N-type semiconductor, resulting in a steady flow of current. In this case, the excess electron holes diffuse as minority carriers in the N-type semiconductor and behave in such a way that the following equation is satisfied (see Section 2.3, Equation (2.30)).

$$\frac{d^2 p_N^*(x)}{dx^2} = \frac{p_N^*(x)}{L_h^2} \tag{3.23}$$

The general solution is given in the following form

$$p_N^*(x) = C_1 \exp\left(-\frac{x}{L_h}\right) + C_2 \exp\left(\frac{x}{L_h}\right) \tag{3.24}$$

Boundary conditions

(I) $p_N^*(\infty) = 0 \longrightarrow C_2 = 0$

(II) $p_N^*(0) = p_N(0) - p_{N0} = p'_P - p_N \longrightarrow C_1 = p'_P - p_N$

From the boundary conditions (I) and (II), equation (3.24) is given by

$$p_N^*(x) = (p'_P - p_N)\exp\left(-\frac{x}{L_h}\right) \tag{3.25}$$

From this, the current density J_h due to electron holes at $x = 0$ is given by

$$J_h = -eD_h \frac{dp_N^*(x)}{dx}\bigg|_{x=0} = \frac{eD_h}{L_h}(p_P' - p_N)\exp\left(-\frac{x}{L_h}\right)\bigg|_{x=0} = \frac{eD_h}{L_h}(p_P' - p_N)$$
$$= \frac{eD_h}{L_h} p_N \left(\exp\left(\frac{eV}{k_B T}\right) - 1\right) \tag{3.26}$$

On the other hand, the conduction electrons (minority carriers) flowing from the N-type semiconductor into the P-type semiconductor can be analyzed in the same way, and the current density J_e due to conduction electrons at $x = 0$ is given by

$$J_e = eD_e \frac{dn_P^*(x)}{dx}\bigg|_{x=0} = \frac{eD_e}{L_e} n_P \left(\exp\left(\frac{eV}{k_B T}\right) - 1\right) \tag{3.27}$$

Assuming that the only current flowing through the PN junction is the diffusion current described above (there is no drift current), the total current density J is given by

$$J = J_h + J_e = J_S \left(\exp\left(\frac{eV}{k_B T}\right) - 1\right) \tag{3.28}$$

where $J_S = e\left(\frac{D_h}{L_h} p_N + \frac{D_e}{L_e} n_P\right)$

Although the current at $x = 0$ is obtained here, the continuity of the current indicates that the same current flows at other arbitrary points. For forward bias ($V \geq 0$), the current density J increases exponentially as the voltage V increases.

(b) In the case of reverse bias ($V \leq 0$)

Next, consider the case where the applied voltage is $V \leq 0$. Figure 3.9 shows the energy band structure when reverse bias is applied. The figure also shows the relationship between ΔE^0, ΔE_V, and ΔE_T. In this case, a large energy barrier is created at the junction.

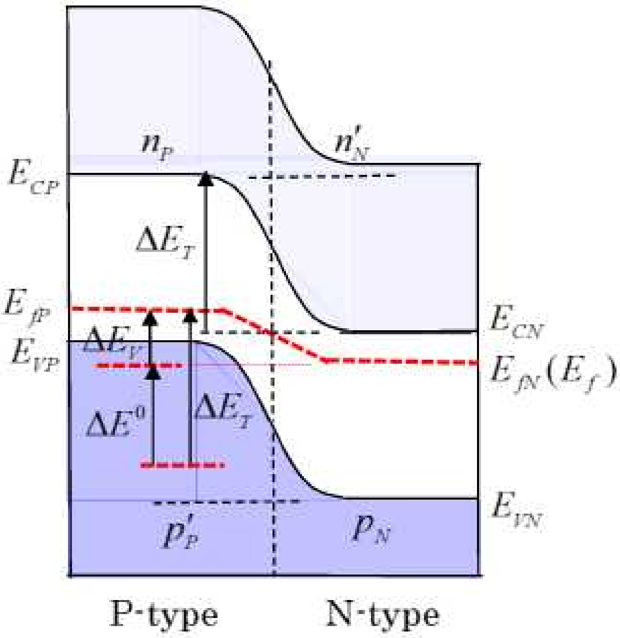

Figure 3.9 Energy band structure when reverse bias ($V \leq 0$) is applied

First, consider the electron hole flow in this case. The electron hole density p_P' ($= p_N\left(1-\exp\left(\frac{eV}{k_BT}\right)\right)$) below the energy level E_{VN} in the P-type semiconductor is smaller than the electron hole density p_N, which is the minority carrier in the N-type semiconductor. Therefore, in this case, $p_N - p_P'$ ($= p_N\left(1-\exp\left(\frac{eV}{k_BT}\right)\right)$) electron holes flow from the N-type semiconductor to the P-type semiconductor. In this case, electron holes supplied from the ground side move through the N-type semiconductor by diffusion. On the other hand, electron holes that flow into the P-type semiconductor dissolve there as part of majority carriers. In this case, the current density J_h due to electron holes at $x = 0$ is given by

$$J_h = -eD_h \left.\frac{dp_N^*(x)}{dx}\right|_{x=0} = \frac{eD_h}{L_h} p_N \left(\exp\left(\frac{eV}{k_B T}\right) - 1 \right) \tag{3.29}$$

Next, consider the conduction electron flow in this case. Similarly, conduction electrons, which are minority carriers of the P-type semiconductor, flow into the N-type semiconductor side, and the current density J_e due to this is as follows.

$$J_e = eD_e \left.\frac{dn_P^*(x)}{dx}\right|_{x=0} = \frac{eD_e}{L_e} n_P \left(\exp\left(\frac{eV}{k_B T}\right) - 1 \right) \tag{3.30}$$

From this, the total current density J is given by

$$J = J_h + J_e = J_S \left(\exp\left(\frac{eV}{k_B T}\right) - 1 \right) \tag{3.31}$$

$$\text{where} \quad J_S = e\left(\frac{D_h}{L_h} p_N + \frac{D_e}{L_e} n_P \right)$$

Although the current at $x = 0$ was obtained here, the same current flows at other arbitrary points due to the continuity of the current, as in the case of forward bias. Equation (3.31) for the reverse bias case ($V \leq 0$) is identical to equation (3.28) for the forward bias case ($V \geq 0$) except for the sign of the voltage V. From this, equation (3.31) (= equation (3.28)) can be used for the current density J regardless of the positive or negative bias voltage V ($V \geq 0$ for forward bias and $V \leq 0$ for reverse bias).

Figure 3.10 shows the current-voltage static characteristics. The current-voltage characteristic exhibits a rectifying characteristic (diode characteristic) and is thus called a PN junction diode. Figure 3.11 shows the small-signal, high-frequency equivalent circuit of a PN junction diode (with unit cross section). The diode part in the figure operates with the current-voltage characteristic shown in Figure 3.10, and C_0 is the PN junction capacitance derived in Section 3.1. In addition, the parallel conductance G_P due to leakage current, the series resistance R_S and the inductance L_S due to the electrode part are included as parasitic components.

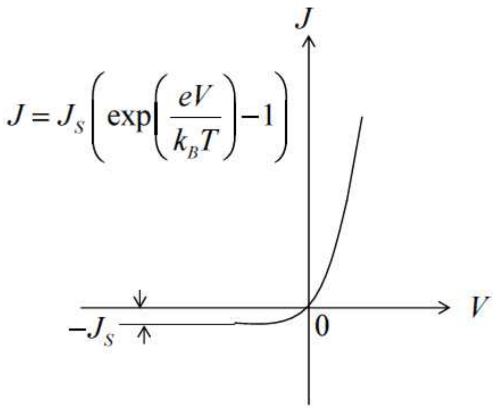

Figure 3.10 Current-voltage static characteristics of a PN junction diode.

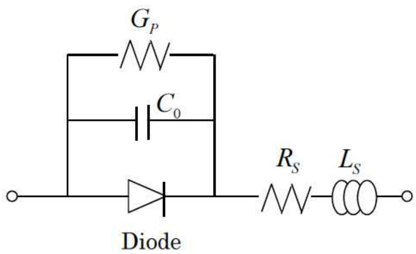

Figure 3.11 Small-signal high-frequency equivalent circuit of a PN junction diode

3.3 PN Junction FET (JFET)

A field effect transistor (FET) is a transistor (three-terminal semiconductor device) that operates by controlling the drift current due to majority carriers flowing in the semiconductor with a gate voltage. Depending on the gate configuration, there are JFET; PN Junction FET, described in this section, and MESFET; Metal-Semiconductor FET, MOSFET; Metal-Oxide-Semiconductor FET, HFET; Heterojunction FET, which are described in later sections.

The PN junction FET (JFET) discussed in this section is the structure used when the FET was first proposed by Shockley et al. Although rarely used today, it is used as the basic structure to explain the operation of FETs. Here we describe an n-channel JFET that uses a Si semiconductor and conduction electrons as carriers (when the carriers are conduction electrons, the JFET is called an n-channel; when the carriers are electron holes, the JFET is called a p-channel). Figure 3.12 shows the structure and operating model of an n-channel JFET. The case of a planar structure is shown for comparison with other FETs described in later sections.

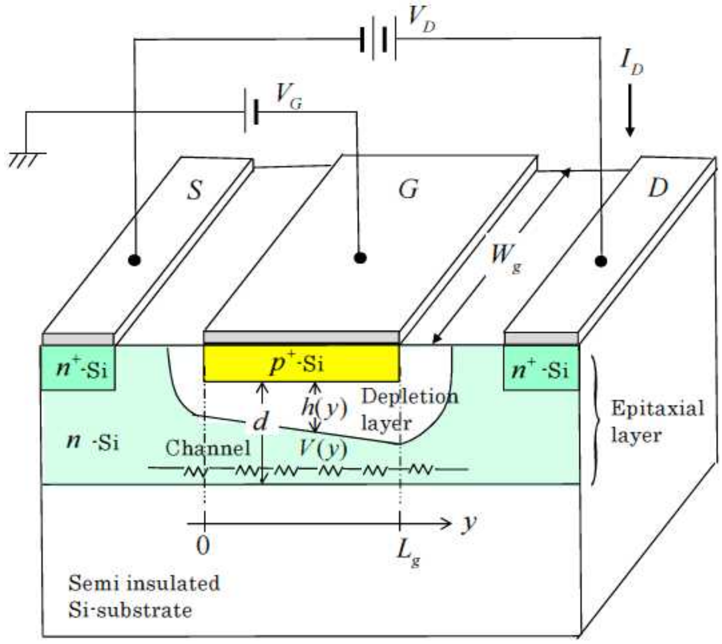

Figure 3.12 PN Junction FET (JFET) Structure and Operation Model (n-channel case)

As shown in the figure, a FET consists of a source (S), drain (D), and gate (G) terminals in an epitaxial Si layer on a semi-insulating Si substrate. The S and D terminals are connected to an N-type semiconductor (n-Si) (actually connected via a high concentration N-type semiconductor ($n^+ - Si$) to reduce resistance when connecting electrodes), and the G terminal is connected to a high concentration P-type semiconductor ($p^+ - Si$). When a voltage is applied between D and S, a drift current due to conduction electrons flows. A depletion layer is formed at the PN junction, and conduction electrons flow through the channel where no depletion layer is formed. Controlling the voltage applied between G and S changes the depletion layer spread, which in turn controls the current flowing between S and D.

The depletion layer spread due to the PN junction at thermal equilibrium was derived in Section 3.1. In the following, the current I_D through the channel is obtained when a small voltage V_D (≥ 0) is applied between D and S and a voltage V_G (≤ 0) is applied between G and S. The analysis here is performed using Shockley's method (Shockley model). When current flows through the channel, the resistance in the channel causes a voltage drop. Take the coordinates as shown in the figure, and assume that the channel voltage $V(y)$ at the coordinate y point is $V(0)=0$ and $V(L_g)=V_D$ (however, the voltage drop in $y \leq 0$ and $L_g \leq y$ region can be approximately neglected). The voltage applied to the PN junction at the coordinate y point becomes $\Delta V_0 + V(y) - V_G$. In this case, the depletion layer spread $h(y)$ in the N-shaped semiconductor at point y is given by (see Equation (3.13))

$$h(y)=\sqrt{\frac{2\varepsilon_S N_A(\Delta V_0+V(y)-V_G)}{eN_D(N_D+N_A)}} \tag{3.32}$$

If the impurity density is chosen to be $N_A >> N_D$, then $h(y)$ is further reduced to

$$h(y)=\sqrt{\frac{2\varepsilon_S(\Delta V_0+V(y)-V_G)}{eN_D}} \tag{3.33}$$

In the following, we assume $N_A >> N_D$ and proceed with the analysis using $h(y)$ given by the above equation (3.33). The thickness of the channel at point y is $d-h(y)$, and the current $I_D(y)$ flowing through it is given by

$$I_D(y)=I_D=eN_D\left(d-h(y)\right)W_g\mu_e\frac{dV(y)}{dy} \tag{3.34}$$

where W_g is the gate width and μ_e is the electron mobility.

Here, the direction of the current $I_D(y)$ is such that the direction of flow through the channel from D to S is positive, and $I_D(y) = I_D$ (a constant value independent of location) due to the continuity of the current. Since $V(y)$ changes $0 \to V_D$ when y changes $0 \to L_g$, equation (3.34) can be expressed by the following integral equation

$$\int_0^{L_g} I_D dy = e\mu_e N_D W_g \int_0^{V_D} \left(d - h(y)\right) dV(y)$$
$$= e\mu_e N_D W_g \int_0^{V_D} \left(d - \left(\frac{2\varepsilon_S \left(V(y) + \Delta V_0 - V_G\right)}{eN_D} \right)^{\frac{1}{2}} \right) dV(y) \tag{3.35}$$

Solving above equation (3.35), the current I_D can be obtained as follows

$$I_D = g_{m0} \left(V_D - \frac{2}{3} \left(\frac{2\varepsilon_S}{eN_D d^2} \right)^{\frac{1}{2}} \left(\left(V_D + \Delta V_0 - V_G\right)^{\frac{3}{2}} - \left(\Delta V_0 - V_G\right)^{\frac{3}{2}} \right) \right) \tag{3.36}$$

$$g_{m0} = e\mu_e N_D d \frac{W_g}{L_g} \tag{3.37}$$

Equation (3.36) gives the current characteristic when the drain voltage V_D is small. As V_D increases, $V(y)$ also increases. From equation (3.33) for $h(y)$, $h(L_g) = d$ at the point $y = L_g$ when $V(y)$ increases and when the gate voltage V_G increases in the negative direction. That is, the channel disappears. This state is called pinch-off, and the drain voltage at this time is called the pinch-off voltage V_P, which is given by

$$h(L_g) = d = \left(\frac{2\varepsilon_S \left(V_P + \Delta V_0 - V_G\right)}{eN_D} \right)^{\frac{1}{2}}$$

$$\longrightarrow \quad V_P = \frac{ed^2 N_D}{2\varepsilon_S} - \Delta V_0 + V_G \tag{3.38}$$

V_P becomes smaller as V_G becomes deeper (larger in the negative direction). When V_D is $V_P \leq V_D$, the excess voltage is used to expand the depletion layer after the channel disappears, and the shape of the channel under the gate remains unchanged as an approximation. From this, I_D becomes a constant value when $V_P \leq V_D$ (Shockley model approach). Substituting V_P in Equation (3.38) for V_D in Equation (3.36), the current when $V_P \leq V_D$ is given as follows.

$$I_D = \frac{1}{3} g_{m0} V_{P0} \left[1 - \frac{3(\Delta V_0 - V_G)}{V_{P0}} + 2\left(\frac{\Delta V_0 - V_G}{V_{P0}} \right)^{\frac{3}{2}} \right] \tag{3.39}$$

where $V_{P0} = \dfrac{eN_D d^2}{2\varepsilon_S}$

From this, the mutual conductance g_m when $V_P \leq V_D$ is given by

$$g_m = \frac{\partial I_D}{\partial V_G} = g_{m0} \left(1 - \left(\frac{\Delta V_0 - V_G}{V_{P0}} \right)^{\frac{1}{2}} \right) \qquad (\leq g_{m0}) \tag{3.40}$$

Figure 3.13 shows schematically the $I_D - V_D$ static and g_m characteristics of the JFET drawn based on the above. The figure also shows the relationship with V_P.

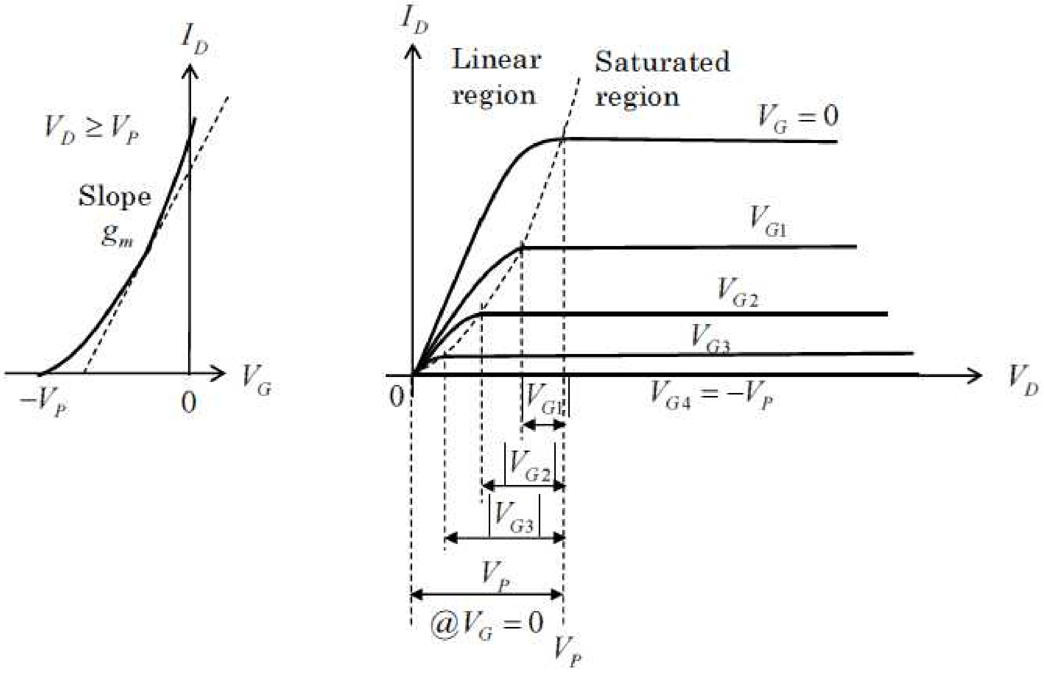

Figure 3.13. $I_D - V_D$ static and g_m characteristics

Assume that all depletion layer capacity at the PN junction when the channel is pinching off (saturated operation) is approximately counted as the G-S junction capacitance C_{GS}. Assuming that the depletion layer spread (thickness) is $d/2$ on average, C_{GS} is approximately given by

$$C_{GS} = \frac{\varepsilon_S L_g W_g}{d/2} \tag{3.41}$$

Figure 3.14 shows the RF equivalent circuit of a JFET when mutual conductance g_m and junction capacitance C_{GS} are considered (other parasitic elements are ignored). In the equivalent circuit shown in the figure, the current gain β for an RF signal at frequency f is given by

$$\beta = \frac{g_m}{j2\pi f C_{GS}} \tag{3.42}$$

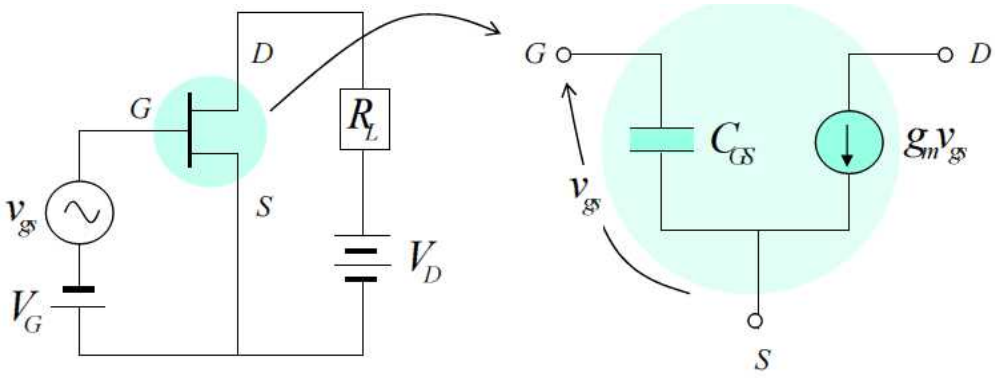

Figure 3.14 RF equivalent circuit of a JFET

From this, the (absolute) maximum value β_{max} of β is given below, using equations (3.37) and (3.41), assuming that the maximum value of g_m is approximately g_{m0}.

$$\beta_{\max} = \frac{g_{m0}}{2\pi f C_{GS}} = \frac{e\mu_e N_D d^2}{4\pi f \varepsilon_S L_g{}^2} \tag{3.43}$$

From this, the maximum cutoff frequency f_{Tmax} at which $\beta_{\max} = 1$ is given by

$$f_{T\max} = \frac{g_{m0}}{2\pi C_{GS}} = \frac{e\mu_e N_D d^2}{4\pi \varepsilon_S L_g{}^2} \tag{3.44}$$

To obtain a high f_{Tmax}, (1) a short gate length L_g, (2) a large electron mobility μ_e, and (3) a large $N_D d^2$ (large amount of carriers in the channel) are required. For (1), L_g is shortened by miniaturization; for (2), compound semiconductors such as GaAs with large electron mobility and heterojunctions are adopted, as described below; and for (3), a configuration to increase the amount of carriers flowing through the channel is adopted.

3.4 Bipolar Transistor (BJT)

There are two types of Bipolar Transistor (BJT; Bipolar Junction Transistor), PNP type and NPN type. The BJT consists of an emitter (E; Emitter), base (B; Base), and collector (C; Collector) region. In the case of the PNP type, each region is a P, N, and P type semiconductor, and in the case of the NPN type, each region is an N, P, and N type semiconductor. This section discusses the NPN-type BJT as an example and describes its operation. Figure 3.15 shows the configuration and operating model of the BJT (NPN type).

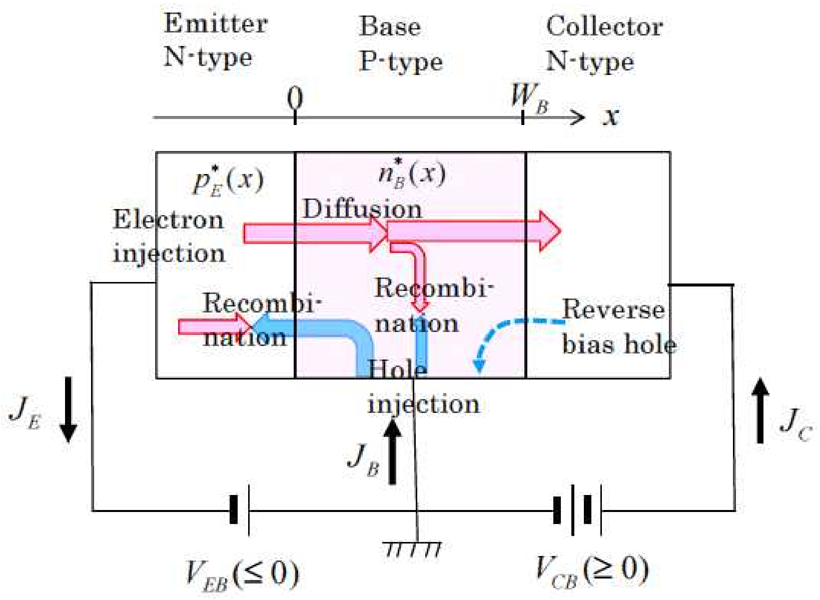

Figure 3.15 Bipolar transistor (BJT) configuration and operating model (for NPN type)

The emitter-to-base bias voltage V_{EB} is the forward voltage of the PN junction ($V_{EB} \leq 0$ for NPN type) and the collector-to-base bias voltage V_{CB} is the reverse voltage of the PN junction ($V_{CB} \geq 0$ for NPN type). The forward bias voltage $V_{EB}(\leq 0)$ between emitter and base injects conduction electrons ("majority carriers" in the emitter and "minority carriers" in the base) from the emitter to the base and electron holes ("majority carriers" in the base and "minority carriers" in the emitter) from the base to the emitter.

(a) Conduction electrons injected into the base diffuse through the base, and some of them recombine with electron holes (majority carriers) in the base, but when the base thickness W_B is sufficiently small compared to the conduction electron diffusion distance L_e. most of the injected conduction electrons reach the collector region without recombining. The conduction electrons arriving at the collector region flow through the collector as a drift current due to the electric field generated by the reverse bias voltage $V_{CB}(\geq 0)$ between the collector and the base.

(b) Electron holes injected into the emitter diffuse through the emitter and recombine with conduction electrons ("majority carriers") in the emitter. In this case, all injected electron holes recombine in the emitter.

(c) On the other hand, a small amount of electron holes (minority carriers in the collector and majority carriers in the base) are injected from the collector to the base by the reverse bias voltage $V_{CB}(\geq 0)$ between the collector and the base, and flow as a reverse current of the diode.

The current will flow due to the movement of carriers above. Figure 3.16 shows the currents in a BJT (NPN type).

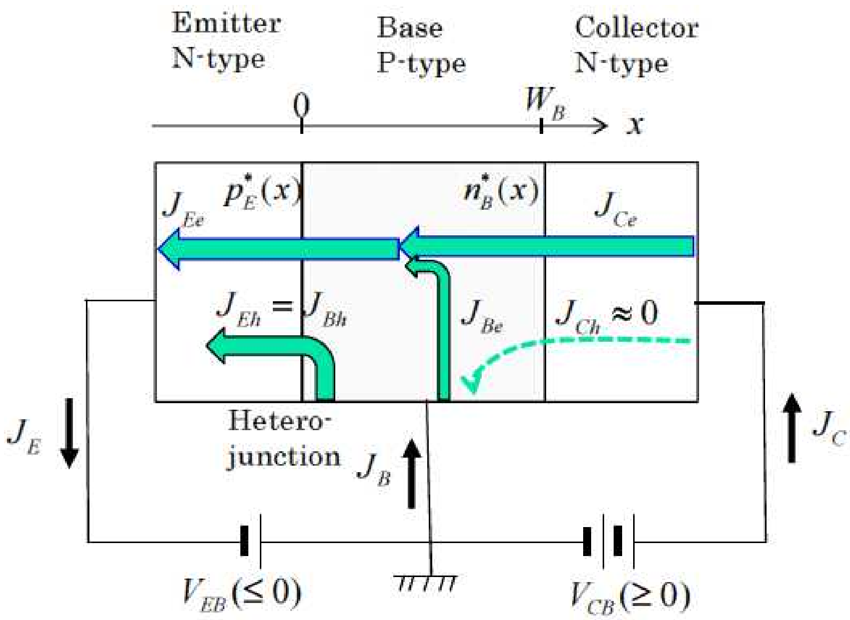

Figure 3.16 Current in a BJT (NPN type)

In the figure, each current represents the following.

J_{Ee}; Current due to conduction electrons injected from the emitter into the base

J_{Ce}; Current due to conduction electrons diffusing through the base and reaching the collector

J_{Be}; Recombination current that flows due to the recombination of conduction electrons injected from the emitter into the base and electron holes in the base
where $J_{Ee} = J_{Ce} + J_{Be}$

J_{Bh}; Current due to electron holes injected into the emitter from the base

J_{Eh}; Recombination current that flows due to the recombination of electron holes injected into the emitter from the base and conduction electrons in the emitter
where $J_{Bh} = J_{Eh}$

J_{Ch}; Reverse current in the diode due to a small amount of electron holes injected into the base from the collector
where $J_{Ch} \approx 0$

J_E; Emitter current (density) $J_E = J_{Ee} + J_{Eh}$

J_B; Base current (density) $J_B = J_{Be} + J_{Bh} - J_{Ch}$

J_C; Collector current (density) $J_C = J_{Ce} + J_{Ch}$

The following relationship holds for the above currents.

$$\begin{aligned} J_E &= J_{Ee} + J_{Eh} = J_{Ce} + J_{Be} + J_{Bh} \\ &= J_{Ce} + J_{Ch} + J_{Be} + J_{Bh} - J_{Ch} = J_C + J_B \end{aligned} \tag{3.45}$$

$$J_B = J_{Bh} + J_{Be} - J_{Ch} \approx J_{Bh} + J_{Be} \tag{3.46}$$

$$J_C = J_{Ce} + J_{Ch} \approx J_{Ce} \tag{3.47}$$

The common base current gain α and the common emitter current gain β are given by

$$\alpha = \frac{J_C}{J_E} \qquad (0 \le \alpha \le 1) \tag{3.48}$$

$$\beta = \frac{J_C}{J_B} = \frac{J_C}{J_E - J_C} = \frac{\alpha}{1-\alpha} \tag{3.49}$$

In the following, the current gains α and β are expressed using physical constants, and the conditions for increasing α and β are derived. Excess conduction electrons $n_B^*(x)$ in the base region diffuse as minority carriers through the P-type semiconductor and behave so as to satisfy the following equation (see Section 2.3, Equation (2.30)).

$$0 = n_B^*(x) - L_e^2 \frac{d^2 n_B^*(x)}{dx^2} \tag{3.50}$$

$$n_B^*(x) = n_B(x) - n_{B0} \tag{3.51}$$

where n_{B0} is the conduction electron density at thermal equilibrium in the base region (P-type semiconductor) and L_e is the conduction electron diffusion distance.

The general solution of equation (3.50) is given by

$$n_B^*(x) = A \exp\left(\frac{-x}{L_e}\right) + B \exp\left(\frac{x}{L_e}\right) \tag{3.52}$$

From the boundary conditions,

$$(\mathrm{I}) \quad n_B^*(0) = n_{B0}(\exp\left(\frac{-eV_{EB}}{k_B T}\right) - 1), \qquad (\mathrm{II}) \quad n_B^*(W_B) = n_{B0}(\exp\left(\frac{-eV_{CB}}{k_B T}\right) - 1)$$

equation (3.52) becomes

$$n_B^*(x)=\frac{n_B^*(0)\sinh\left(\dfrac{W_B-x}{L_e}\right)+n_B^*(W_B)\sinh\left(\dfrac{x}{L_e}\right)}{\sinh\left(\dfrac{W_B}{L_e}\right)} \tag{3.53}$$

Here, J_{Ee} (the current due to conduction electrons injected from the emitter to the base) is the diffusion current due to conduction electrons at $x = 0$, and is given by

$$J_{Ee}=-eD_e\frac{dn_B^*(x)}{dx}\bigg|_{x=0}=\frac{eD_e}{L_e}\coth\left(\frac{W_B}{L_e}\right)\left(n_B^*(0)-n_B^*(W_B)\frac{1}{\cosh\left(W_B/L_e\right)}\right)$$
$$=\frac{eD_en_{B0}}{L_e}\coth\left(\frac{W_B}{L_e}\right)\left((\exp\left(\frac{-eV_{EB}}{k_BT}\right)-1)-\frac{1}{\cosh\left(W_B/L_e\right)}(\exp\left(\frac{-eV_{CB}}{k_BT}\right)-1)\right) \tag{3.54}$$

Further approximations using the condition $W_B << L_e$ yield the following equation.

$$J_{Ee}\approx\frac{eD_en_{B0}}{W_B}\left((\exp\left(\frac{-eV_{EB}}{k_BT}\right)-1)(1+\frac{1}{2}\left(\frac{W_B}{L_e}\right)^2)-(\exp\left(\frac{-eV_{CB}}{k_BT}\right)-1)\right) \tag{3.55}$$

Similarly, J_{Ce} (the current due to conduction electrons diffusing through the base and reaching the collector) is the diffusion current due to conduction electrons at $x = W_B$ and is given by

$$J_{Ce}=-eD_e\frac{dn_B^*(x)}{dx}\bigg|_{x=W_B}=\frac{-eD_e}{L_e\sinh\left(W_B/L_e\right)}\left(-n_B^*(0)\cosh\left(\frac{W_B-x}{L_e}\right)+n_B^*(W_B)\cosh\left(\frac{x}{L_e}\right)\right)$$
$$=\frac{eD_en_{B0}}{L_e\sinh\left(W_B/L_e\right)}\left((\exp\left(\frac{-eV_{EB}}{k_BT}\right)-1)+\cosh\left(\frac{x}{L_e}\right)(1-\exp\left(\frac{-eV_{CB}}{k_BT}\right))\right) \tag{3.56}$$

Further approximating with the condition $W_B << L_e$, we obtain the following equation.

$$J_{Ce} \approx \frac{eD_e n_{B0}}{W_B}\left((\exp\left(\frac{-eV_{EB}}{k_B T}\right) - 1) + (1 + \frac{1}{2}\left(\frac{W_B}{L_e}\right)^2)(1 - \exp\left(\frac{-eV_{CB}}{k_B T}\right)) \right) \tag{3.57}$$

On the other hand, excess electron holes $p_E^*(x)$ in the emitter region diffuse through the emitter region (N-type semiconductor) as minority carriers and operate to satisfy the following equation.

$$0 = p_E^*(x) - L_h^2 \frac{d^2 p_E^*(x)}{dx^2} \tag{3.58}$$

$$p_E^*(x) = p_E(x) - p_{E0} \tag{3.59}$$

where p_E is the electron hole density at thermal equilibrium in the emitter region (N-type semiconductor) and L_h is the electron hole diffusion distance.

The general solution of equation (3.58) is given by

$$p_E^*(x) = A' \exp\left(\frac{-x}{L_h}\right) + B' \exp\left(\frac{x}{L_h}\right) \tag{3.60}$$

From the boundary conditions,

(I) $p_E^*(-\infty) = 0$, (II) $p_E^*(0) = p_{E0}(\exp\left(\frac{-eV_{EB}}{k_B T}\right) - 1)$

equation (3.60) becomes

$$p_E^*(x) = p_{E0}(\exp\left(\frac{-eV_{EB}}{k_B T}\right) - 1)\exp\left(\frac{x}{L_h}\right) \tag{3.61}$$

$J_{Eh}(=J_{Bh})$ (the current due to electron holes injected from the base into the emitter) is given as the diffusion current due to electron holes at $x = 0$ by

$$\begin{aligned} J_{Bh} = J_{Eh} = eD_h \left.\frac{dp_E^*(x)}{dx}\right|_{x=0} &= eD_h \frac{p_{E0}}{L_h}(\exp\left(\frac{-eV_{EB}}{k_B T}\right) - 1)\exp\left.\left(\frac{x}{L_h}\right)\right|_{x=0} \\ = eD_h \frac{p_{E0}}{L_h}(\exp\left(\frac{-eV_{EB}}{k_B T}\right) - 1) \end{aligned} \tag{3.62}$$

Here, when approximating $\exp\left(\frac{-eV_{CB}}{k_B T}\right) \approx 0$ and further assuming

$\exp\left(\frac{-eV_{EB}}{k_B T}\right) - 1 = \theta(>>1)$, J_{Ee} , J_{Ce} , and $J_{Bh} = J_{Eh}$ are expressed as follows.

$$J_{Ee} = \frac{eD_e n_{B0}}{W_B}\left(\theta + 1 + \frac{1}{2}\theta\left(\frac{W_B}{L_e}\right)^2\right) \tag{3.63}$$

$$J_{Ce} = \frac{eD_e n_{B0}}{W_B}\left(\theta + 1 + \frac{1}{2}\left(\frac{W_B}{L_e}\right)^2\right) \tag{3.64}$$

$$J_{Bh} = J_{Eh} = \frac{eD_h p_{E0}}{L_h}\theta \tag{3.65}$$

Using equations (3.45)-(3.47) and (3.63)-(3.65), the current gains α and β are given by

$$\alpha = \frac{J_C}{J_E} = \frac{J_{Ce}}{J_{Ee} + J_{Eh}} = \frac{\gamma_1}{1 + \gamma_2} \tag{3.66}$$

$$\beta = \frac{J_C}{J_B} = \frac{J_{Ce}}{J_{Be} + J_{Bh}} = \frac{J_{Ce}}{J_{Be} + J_{Eh}} = \frac{J_{Ch}}{J_{Ee} - J_{Ce} + J_{Eh}} = \frac{\gamma_1}{1 - \gamma_1 + \gamma_2} \tag{3.67}$$

where

$$\gamma_1 = \frac{J_{Ce}}{J_{Ee}} = \frac{\theta + 1 + \frac{1}{2}\left(\frac{W_B}{L_e}\right)^2}{\theta + 1 + \frac{1}{2}\theta\left(\frac{W_B}{L_e}\right)^2} \qquad (0 \le \gamma_1 \le 1) \tag{3.68}$$

$$\gamma_2 = \frac{J_{Eh}}{J_{Ee}} = \frac{\theta}{\theta + 1 + \frac{1}{2}\theta\left(\frac{W_B}{L_e}\right)^2} \frac{W_B}{D_e n_{B0}} \frac{D_h p_{E0}}{L_h} \tag{3.69}$$

Furthermore, since $\theta >> 1$

$$\gamma_1 \approx \frac{1}{1 + \frac{1}{2}\left(\frac{W_B}{L_e}\right)^2} \qquad (0 \le \gamma_1 \le 1) \tag{3.70}$$

$$\gamma_2 \approx \frac{1}{1 + \frac{1}{2}\left(\frac{W_B}{L_e}\right)^2} \frac{W_B}{D_e n_{B0}} \frac{D_h p_{E0}}{L_h} \tag{3.71}$$

From equations (3.66) and (3.67), we can say that to make α large or β large, γ_1 should be large and γ_2 should be small. To make γ_1 large, $\frac{W_B}{L_e}$ must be small, i.e., the base thickness W_B must be sufficiently small compared to the diffusion distance L_e. In considering γ_2, by approximating $D_h \approx D_e$ and $L_h \approx L_e$, and using the constant np product relationship, equation (3.71) becomes

$$\gamma_2 \approx \frac{\dfrac{W_B}{L_e}}{1+\dfrac{1}{2}\left(\dfrac{W_B}{L_e}\right)^2}\frac{p_{B0}}{n_{E0}} \tag{3.72}$$

From this, to make γ_2 small, the base thickness W_B must be smaller than the diffusion distance $L_e(\approx L_h)$, and p_{B0}/n_{E0} must be smaller. Since $n_{E0} \approx N_{D,E}$ ($N_{D,E}$ is the donor density in the emitter) and $p_{B0} \approx N_{A,B}$ ($N_{A,B}$ is the acceptor density in the base) at room temperature, the impurity (donor) concentration $N_{D,E}$ in the emitter region should be sufficiently larger than the impurity (acceptor) concentration $N_{A,B}$ in the base region. Alternatively, to reduce γ_2, the emitter-to-base PN junction can be made heterojunction and $J_{Eh} \leq J_{Ee}$ (heterojunction bipolar transistor (HBT) as described below).

Figure 3.17 shows an example configuration of an NPN-type BJT using Si. To obtain large current gains α and β, the base thickness dimension W_B is fabricated to be sufficiently narrow compared to the diffusion distance $L_e \approx L_h$, and a n^+-type semiconductor with a high impurity (donor) concentration is used in the emitter region.

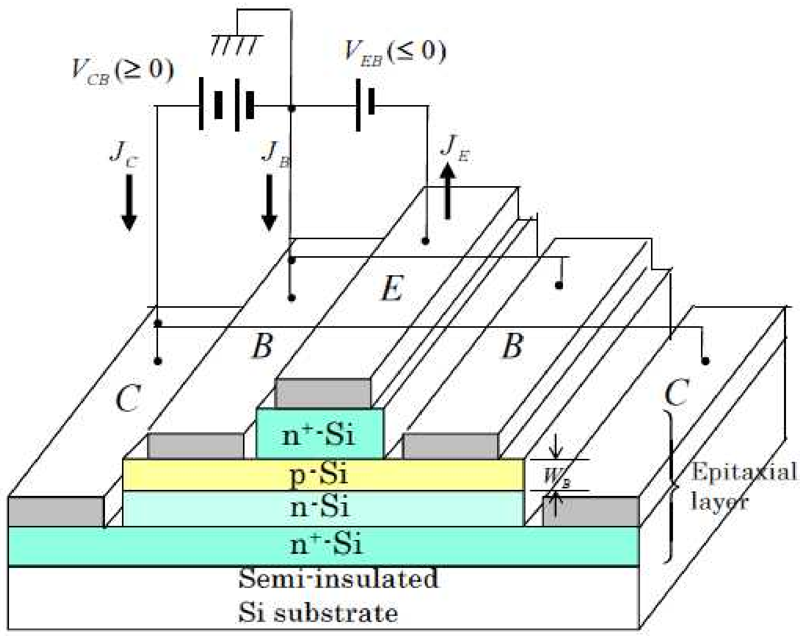

Figure 3.17 Composition example of a BJT (NPN type)

Figure 3.18 shows the DC and RF equivalent circuits of a BJT. The figure shows the case where a NPN-type BJT is used with emitter grounding.

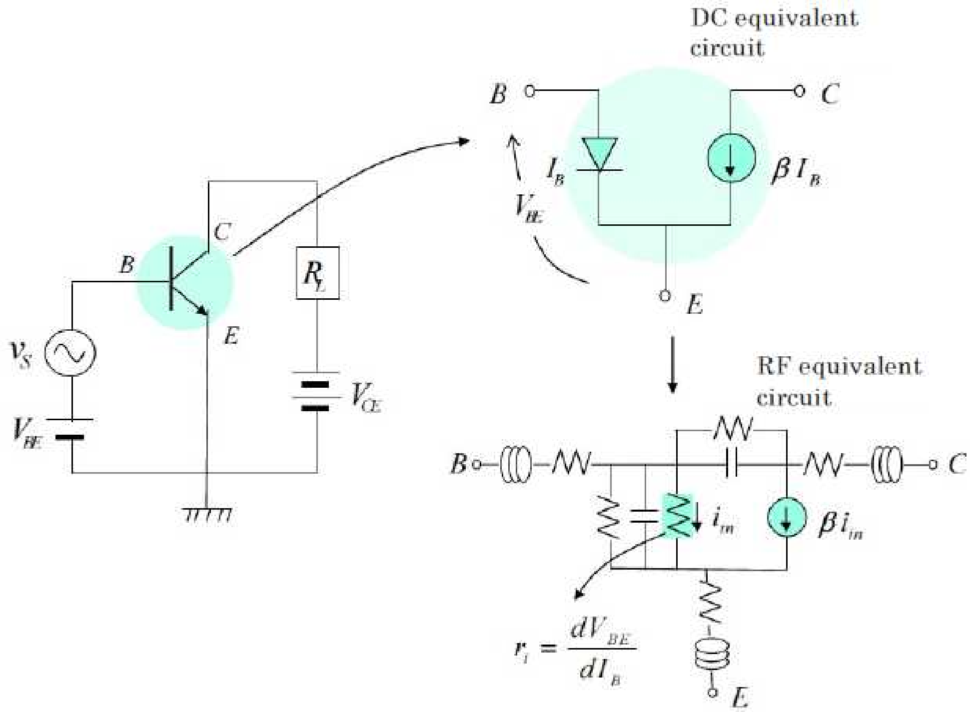

Figure 3.18 DC and RF equivalent circuits of a BJT (NPN type)

Appendix E　Relationship between Potential Energy, Potential, Electric Field and Force

Figure E.1 shows the electric field $\mathsf{E}(x)$ and force $F(x)$ on a carrier of charge q ($q = -e$ for conduction electrons, $q = e$ for electron holes) in a field of potential V(x).

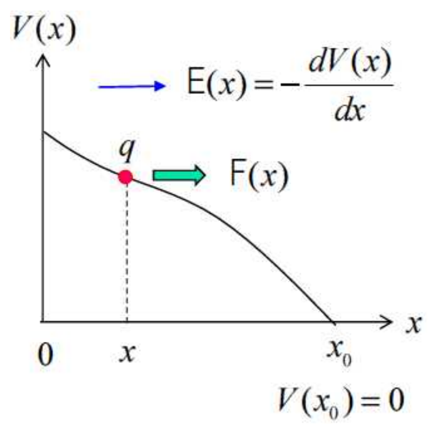

Figure E.1　Electric field $\mathsf{E}(x)$ and force $F(x)$ on a carrier of charge q in a field of potential $V(x)$

The electric field $\mathsf{E}(x)$ at x has the following relationship to the potential $V(x)$.

$$\mathsf{E}(x) = -\frac{dV(x)}{dx} \tag{E.1}$$

where the force $F(x)$ on the charge q is

$$\mathsf{F}(x) = q\mathsf{E}(x) = -q\frac{dV(x)}{dx} \tag{E.2}$$

The potential energy $E_p(x)$ of a carrier at x is defined as the amount of work done by the carrier during its slow (velocityless) movement (no kinetic energy) from a point at x to infinity ($x_0 \to \infty$, $V(x_0) = 0$). From this, the potential energy $E_P(x)$ can be expressed using the potential $V(x)$ as

$$
\begin{aligned}
E_P(x) &= \int_x^{x_0} \mathrm{F}(x)dx = \int_x^{x_0} -q\frac{dV(x)}{dx}dx = -qV(x)\Big|_x^{x_0} \\
&= -q\left[V(x_0) - V(x)\right] = qV(x)
\end{aligned}
\tag{E.3}
$$

This equation expresses the potential energy $E_P(x)$ possessed by a carrier of charge q placed in a field whose potential is $V(x)$. Conversely, it can be said to express the potential $V(x)$ of the field in which the carrier is placed when the potential energy is $E_P(x)$.

When the potential energy is at the Fermi levels of E_{fP} and E_{fN}, the potentials of V_{fP} and V_{fN} there are given by

$$
E_{fP} = -eV_{fP} \qquad\qquad E_{fN} = -eV_{fN} \tag{E.4}
$$

When the potential energy of an electron is the energy level E_C at the bottom of the conduction band (the energy at the bottom of the conduction band has zero kinetic energy, and E_C means potential energy), and E_C is a function of x, from Equation (E.3), the potential $V(x)$ of an electron at x is expressed as follows。

$$
E_C(x) = -eV(x) \tag{E.5}
$$

At this time, from Equation (E.2), the following relationship is obtained

$$
\mathrm{F}(x) = -e\mathrm{E}(x) = \frac{d(eV(x))}{dx} = -\frac{dE_C(x)}{dx} \tag{E.6}
$$

Appendix F　Derivation of Equation (3.18)

In the figure F.1, the conduction electron density n_N' in an N-type semiconductor at an energy level higher than the energy level E_{CP} at the bottom of the conduction band of a P-type semiconductor is given by

$$\begin{aligned} n'_N &= N_C \exp\left(-\frac{E_{CP}-E_f}{k_B T}\right) \\ &= N_C \exp\left(-\frac{E_{CN}-E_f}{k_B T}\right)\exp\left(-\frac{E_{CP}-E_{CN}}{k_B T}\right) = n_N \exp\left(-\frac{\Delta E^0}{k_B T}\right) \end{aligned} \tag{F.1}$$

Where n_N is the conduction electron density of the N-type semiconductor at thermal equilibrium, and is given by the following equation from Section 1.5, Equation (1.17)

$$n_N = N_C \exp\left(-\frac{E_{CN}-E_f}{k_B T}\right) \tag{F.2}$$

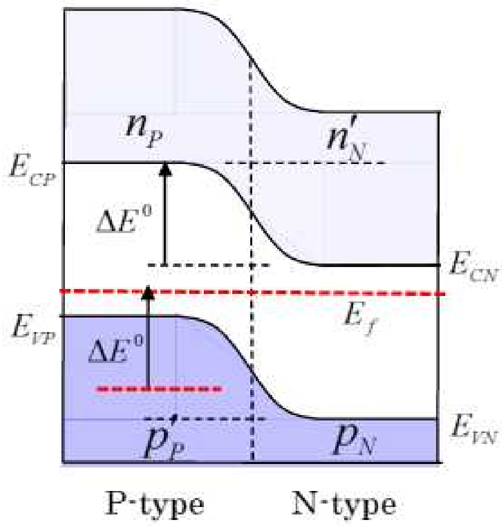

Figure F.1 Energy band structure at no bias in PN junction

In thermal equilibrium, the conduction electron density n_P above energy E_{CP} in the P-type semiconductor is equal to the conduction electron density n'_N above energy E_{CP} in the N-type semiconductor, and no current flows between the junctions. At this time, the following equation holds.

$$n_P = n'_N = n_N \exp\left(-\frac{\Delta E^0}{k_B T}\right) \tag{F.3}$$

Next, consider the case where a bias voltage V is applied (N-type semi-conductor side to ground and P-type semiconductor side to V). Figure F.2 shows the energy band structure when bias is applied. When a bias voltage V is applied, ΔE^0 should be replaced by $\Delta E_T (= \Delta E^0 + \Delta E_V = \Delta E^0 - eV)$ in equation (F.1) for n_N'. Specifically, the following is shown for n_N'.

$$\begin{aligned} n'_N &= n_N \exp\left(-\frac{\Delta E_T}{k_B T}\right) = n_N \exp\left(-\frac{\Delta E^0}{k_B T}\right)\exp\left(-\frac{\Delta E_V}{k_B T}\right) \\ &= n_P \exp\left(-\frac{\Delta E_V}{k_B T}\right) = n_P \exp\left(\frac{eV}{k_B T}\right) \end{aligned} \tag{F.4}$$

Note that equation (F.3) is used here for the conversion between n_N and n_P.

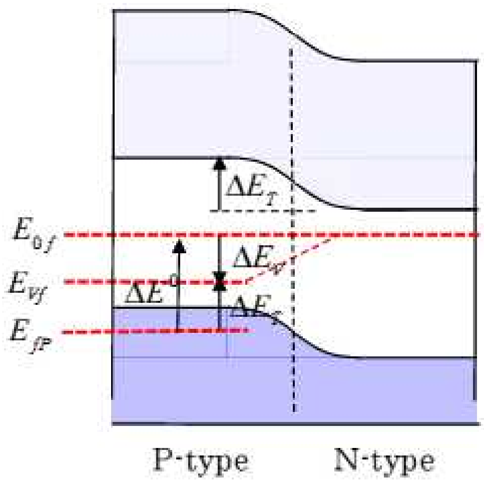

Figure F.2　Energy band structure of PN junction when bias is applied